Unit - I. Force Analysis.

Part - I / Dynamic force analyis

Inertia :

The property of matter offering resistance to any change of it's state of rest or of uniform motion in a straight line is known as inertia.

Inertia force :

The inertia of a body opposes this external force applied (F) and it is known as inertia force.

$$\boxed{\begin{aligned}\text{Inertia force} &= -\text{ external (accelerating) force}\\ &= -m \cdot a\end{aligned}} \quad \text{—} ①$$

Inertia torque :

The inertia of the body opposes this exertal torque applied (T) and it is known as inertia torque.

$$\boxed{\text{Inertia torque} = -\text{ externally applied torque}} \quad \text{—} ②$$

D' Alembert's principle :

It states that, the inertia forces and torques, and the external forces and torques, acting on a body together results in statical equillibrium.

The eqns ① and ② can also be written as

$$F + (-ma) = 0 \text{ —— } ③$$

$$T + (-I\alpha) = 0 \text{ —— } ④$$

The above eqns ③ & ④ are known as D'Alembert's principle.

eqns ③ & ④ can also be written as,

$$\left.\begin{aligned}\Sigma F &= 0\\ \Sigma M &= 0\end{aligned}\right\} \text{—— } ⑤.$$

Application of D' Alembert's principle :

This principle is used to reduce a dynamic analysis problem into an equivalent problem of static equillibrium.

Dynamic Analysis of reciprocating engines:

i) Velocity and Acceleration of reciprocating parts in engine:

The velocity and acceleration of various parts of reciprocating mechanism can be determined both analytically and graphically. The various methods used are

(i) Analytical Method.

(ii) Graphical Method.

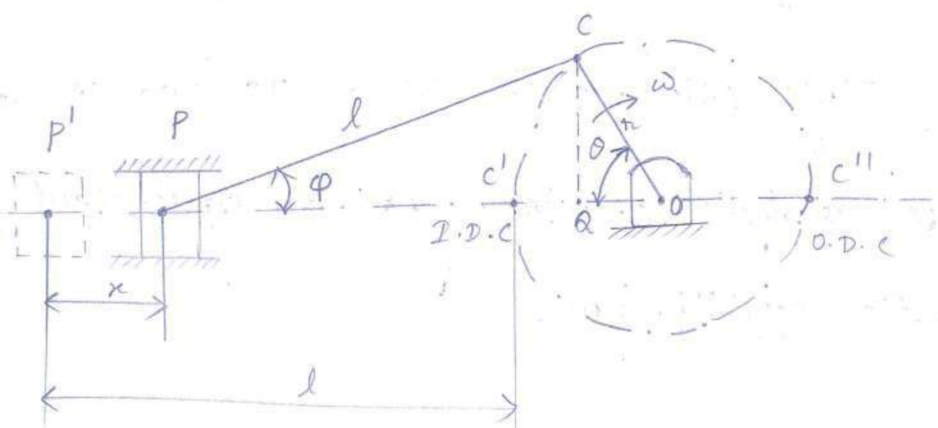
a) Klein's construction.

b) Ritterhaus's construction.

c) Benett's construction.

Analytical Method:

fig: Reciprocating Engine Mechanism.



Let,

$r \rightarrow$ crank radius.

$l \rightarrow$ length of connecting rod.

$\theta \rightarrow$ Angle made crank with I.D.C.

$\varphi \rightarrow$ Inclination of connecting rod to the line of stroke PO.

$n = \frac{l}{r}$ = Ratio of length of connecting rod to the radius of crank, also known as 'obliquity ratio'.

Forces on the reciprocating parts of an engine, neglecting the weight of the connecting rod:

A] To find the net load on the piston (F_L):

(i) For single-cylinder single acting engine:

Let,

$P \rightarrow$ Net pressure of steam (or) gas on the piston in N/m^2.

$D \rightarrow$ Diameter of the piston in 'm'.

Then, Net load on the piston is given by,

F_L = Pressure × Area.

$F_L = P \times \frac{\pi}{4} D^2$.

(ii) For single-cylinder, double-acting engine:

Let,

P_1 and A_1 → Pressure and c/s area on the back end side of the piston resp.

P_2 & A_2 → Pressure and c/s area on the crank end side of the piston resp.

a → c/s area of the piston rod.

$$a = \frac{\pi}{4} d^2.$$

d → dia. of piston rod.

Then, net load on the piston is given by,

$$F_L = P_1 A_1 - P_2 A_2.$$

$$F_L = P_1 A_1 - P_2 (A_1 - a) \quad [\because A_2 = A_1 - a].$$

Formula Used:

Forces on reciprocating parts of an engine:

1) Piston effort (F_p):

a) For horizontal reciprocating engines:

Piston effort, $F_p = F_L \pm F_I$ (neglecting frictional resistance)

$F_p = F_L \pm F_I - R_F$ [considering frictional resistance].

b) For vertical reciprocating engines:

Piston effort, $F_p = F_L \pm F_I \pm W_R$ [neglecting frictional resistance]

$F_p = F_L \pm F_I \pm W_R - R_F$ [considering frictional resistance]

B] To find inertia force on the reciprocating parts: (F_I):

Inertia force, $F_I = m_R a_R = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

(i) Force acting along the connecting rod (F_Q):

$$F_Q = \frac{F_P}{\cos\varphi}$$

(ii) Thrust on the sides of cylinder walls: (F_N):

$$F_N = F_Q \sin\varphi = F_p \tan\varphi.$$

(iii) Crank-pin effort (F_T):

$$F_T = F_Q \sin(\theta + \varphi) = \frac{F_P}{\cos\varphi} \times \sin(\theta+\varphi)$$

(iv) Thrust on crank shaft bearing (F_B):

(v) Crank effort on the crank shaft (T):

$$T = F_T \times r$$

$$= \left[\frac{F_P}{\cos\phi} \sin(\theta + \phi)\right] \times r$$

$$T = F_P \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] \times r$$

Pbms :

1) The length of crank and connecting rod of a horizontal engine are 200mm and 1m resp. The crank is rotating at 400rpm. When the crank has turned through 30° from the inner dead center, the difference of pressure between cover and piston rod is 0.4 N/mm². If the mass of the reciprocating parts is 100kg and cylinder bore is 0.4m, calculate

(i) Inertia force.

(ii) Force (load) on the piston.

(iii) Piston effort.

(iv) Thrust on the sides of the cylinder walls.

(v) Thrust in the connecting rod.

Gn:

$r = 200\,mm = 0.2\,m.$

$l = 1\,m$

$N = 400\,r.p.m.$

$\theta = 30°.$

$P_1 - P_2 = 0.4\,N/mm^2 = 0.4 \times 10^6\,N/m^2.$

$m_R = 100\,kg.$

$D = 0.4\,m.$

Soln:

$$\omega = \frac{2\pi N}{60} = 41.89\ rad/s.$$

$$\text{and, } n = \frac{l}{r} = \frac{1}{0.2} = 5$$

(i) Inertia force (F_I):

$$F_I = m_R\,\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right].$$

$$F_I = 100 \times (41.89)^2 \times 0.2\left[\cos 30° + \frac{\cos 2(30°)}{5}\right]$$

$$= 35095.442\,(0.966)$$

$$F_I = 33903.08$$

$$\Rightarrow F_I = 33.903 \times 10^3\ N.$$

$$F_I = 33.903\ KN.$$

(ii) Net load on the piston (F_L):

$$F_L = (P_1 - P_2)\,A.$$

$$= 0.4 \times 10^6 \times \frac{\pi}{4}(0.4)^2.$$

$= 50.265 \times 10^3$ N.

$F_L = 50.265$ KN.

(iii) Piston effort:

$F_P = F_L - F_I$.

$= 50.265 - 33.903$

$F_P = 16.36$ KN.

(iv) Thrust on sides of the cylinder walls:

$F_N = F_P \tan\varphi$

To find φ:

$\sin\varphi = \frac{\sin\theta}{n} = \frac{\sin 30^\circ}{5} = 0.1$

(or)

$\varphi = 5.74^\circ$.

$\therefore F_N = 16.36 \tan 5.74^\circ$.

$F_N = 1.644$ KN.

(v) Thrust in the connecting rod:

$F_Q = \frac{F_P}{\cos\varphi}$

$= \frac{16.36 \times 10^3}{\cos 5.74^\circ}$

$F_Q = 16.444$ KN.

(vi) Crank effort (T):

W.K.T, tangential force, $F_T = F_Q \sin(\theta + \varphi)$

$= 16.44 \sin(30 + 5.74)$.

$F_T = 9.605$ KN.

W.K.T, crank effort, $T = F_T \times r$.

$= 9.605 \times 0.2$

$T = 1921.13$ N-m.

Result:

(i) Inertia force, $F_I = 33.903$ KN.

(ii) Force on piston, $F_L = 50.265$ KN.

(iii) Piston effort, $F_P = 16.36$ KN.

(iv) Thrust on sides of the cylinder walls, $F_N = 1.644$ KN.

(v) Thrust in the connecting rod, $F_Q = 16.44$ KN.

(vi) Crank effort, $T = 1921.13$ N.m

2) A horizontal steam engine running at 240 rpm has a bore of 300 mm and stroke 600 mm. The connecting rod is 1.25 m long and the mass of reciprocating parts is 60 kg. When the crank is 60° with inner dead center, the steam pressure on the cover side of the piston is 1.125 N/mm² while that

on the crank side is 0.125 N/mm^2.

Neglecting the area of piston rod, determine

a) Force on the piston rod. (Piston effort)

b) Turning moment on the crank shaft. (crank effort)

Gn:

$N = 240$ r.p.m.

$D = 300$ mm $= 0.3$ m.

$L = 600$ mm $= 0.6$ m.

(or)

$r = \frac{L}{2} = \frac{0.6}{2} = 0.3$ m.

$l = 1.25$ m.

$m_R = 60$ kg

$\theta = 60°$

$P_1 = 1.125\ N/mm^2 = 1.125 \times 10^6\ N/m^2$.

$P_2 = 0.125\ N/mm^2 = 0.125 \times 10^6\ N/m^2$.

Soln:

$\omega = \frac{2\pi N}{60} = 25.13$ rad/s.

$n = l/r = \frac{1.25}{0.3} = 4.167$.

a) Force on the piston rod: (F_P):

W.K.T,

$F_L = (1.125 \times 10^6 - 0.125 \times 10^6)\frac{\pi}{4}(0.3)^2$.

$F_L = 70.685$ KN.

Also, Inertia force on reciprocating parts,

$F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$.

$= 60 \times (25.13)^2 \times 0.3 \left[\cos 60° + \frac{\cos 2(60°)}{4.167}\right]$

$F_I = 4.319$ KN.

$\therefore$ Force on piston rod, $F_P = F_L - F_I$.

$= 70.685 - 4.319$.

$F_P = 66.366$ KN.

b) Turning moment on the crankshaft (T):

$T = F_P \times r \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$.

$= 66.366 \times 0.3 \left[\sin 60° + \frac{\sin 2(60°)}{2\sqrt{(4.167)^2 - \sin^2 60°}}\right]$

$= 19.358$ KN-m.

$T = 19358$ N-m.

Result:

(i) Force on piston rod, $F_P = 66.366$ KN.

(ii) Turnin

3) A horizontal steam engine running at 210 rpm has a bore of 190 mm and stroke of 350 mm. The piston rod is 20 mm in diameter and connecting rod length is 950 mm. The mass of the reciprocating parts is 8 kg and the frictional resistance is equivalent to a force of 350 N. Determine the following when the crank is 115° from the inner dead center, the mean pressure being 4500 N/m² on the cover side and 100 N/mm² on the crank side.

(i) Thrust on the connecting rod. $F_Q = \frac{F_P}{\cos\phi}$

(ii) Thrust on the cylinder walls. F_N

(iii) Load on the bearings. F_B

(iv) Turning movement on the crankshaft. T

Gn:

$N = 210$ rpm.

$D = 190$ ~~Af~~ mm $= 0.19$ m.

$L = 350$ mm $= 0.35$ m.

$\therefore r = \frac{L}{2} = \frac{0.35}{2} = 0.175$ m.

$d = 20$ mm $= 0.02$ m

$m_R = 8$ kg

$R_F = 350$ N.

$\theta = 115°$.

$P_1 = 4500$ N/mm²

$P_2 = 100$ N/m

$\omega = \frac{2\pi N}{60} = \frac{2\pi(210)}{60} = 21.99$ rad/s.

$\therefore$ ~~ω~~ $n = \frac{l}{r} = \frac{0.95}{0.175} = 5.43$.

Soln:

Load on piston: (F_L)

W.K.T, for double acting reciprocating engine,

$F_L = P_1 A_1 - P_2 A_2$.

$F_L = P_1 A_1 - P_2 (A_1 - a)$.

Area of piston on cover side, $A_1 = \frac{\pi}{4} D^2$.

$= \frac{\pi}{4}(0.19)^2$.

$A_1 = 0.0284$ mm².

Area of piston rod, $a = \frac{\pi}{4} d^2$

$= \frac{\pi}{4}(0.02)^2$.

$\therefore a = 3.142 \times 10^{-4}$ m².

$\therefore F_L = P_1 A_1 - P_2 (A_1 - a)$

$= 4500(0.0284) - 100(0.0284 - 3.142\times10^{-4})$

$= 127.8 - 2.808$

$F_L = 124.99\ N$

To find inertia force (F_I):

$$F_I = m_R\,\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right].$$

$$= 8\times(21.99)^2\times 0.175\left[\cos 115^\circ + \frac{\cos 2(115^\circ)}{5.43}\right].$$

$= 676.984\ (-0.541)$

$F_I = -366.245\ N$

$\therefore$ Piston effort is given by,

$F_P = F_L - F_I - R_F$

$= 124.99 + 366.245 - 350.$

$F_P = 141.24\ N$

(i) Thrust on connecting rod (F_Q):

$$F_Q = \frac{F_P}{\cos\varphi}$$

here, $\sin\varphi = \dfrac{\sin\theta}{n}.$

$= \dfrac{\sin 115}{5.43}$

$\sin\varphi = 0.1669$

$\Rightarrow \varphi = 9.61^\circ.$

$$\therefore F_Q = \frac{F_P}{\cos\varphi} = \frac{141.24}{\cos 9.61}$$

$F_Q = 143.25\ N.$

(ii) Thrust on the cylinder walls (F_N):

$F_N = F_P \tan\varphi$

$= 141.24 \tan 9.61$

$= 24\ N.$

(iii) Load on bearings (F_B):

$F_B = F_Q(\cos(\theta + \varphi))$

$= 143.25\cos(115 + 9.61)$

$F_B = -81.236\ N.$

(iv) Turning moment on the crankshaft : (T)

$$T = \left[\frac{F_P}{\cos\varphi} \sin(\theta+\varphi)\right] r .$$

$$= \left[\frac{141.24}{\cos 9.61} \sin(115+9.61)\right] 0.175$$

$T = 20.63$ Nm

Result :

(i) Thrust on connecting rod $(F_Q) = 143.25$ N

(ii) Thrust on cylinder wall $(F_N) = 24$ N

(iii) Load on bearings $(F_B) = -81.36$ N

(iv) Turning moment on crankshaft $(T) = 20.63$ Nm

4) A vertical petrol engine 150mm diameter and 200 mm stroke has a connecting rod 350 mm long. The mass of piston is 1.6 kg and the engine speed is 1800 rpm. On the expansion stroke with crank changed 30° from the top dead center, the gas pressure is 750 KN/m². Determine the net

Gn:

$D = 150$ mm $= 0.15$ m.

$L = 200$ mm $= 0.2$ m.

$\therefore r = \frac{L}{2} = \frac{0.2}{2} = 0.1$ m

$m_R = 1.6$ kg .

$l = 350$ mm $= 0.35$ m .

$N = 1800$ r.p.m .

$\theta = 30°$.

$P = 750$ KN/m² $= 750 \times 10^3$ N/m².

Soln:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1800)}{60} = 188.49 \text{ rad/s}$$

$$n = \frac{l}{r} = \frac{0.35}{0.1} = 3.5$$

Load on the piston, $F_L = P \times A$.

$$= P \times \frac{\pi}{4} D^2 .$$

$$= 750 \times 10^3 \times \frac{\pi}{4} (0.15)^2 .$$

$F_L = 13253.6$ N

Inertia force, $F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$.

$$= (1.6)(188.49)^2 (0.1)\left[\cos 30° + \frac{\cos 2(30)}{3.5}\right]$$

$F_I = 5735.05$ N .

For vertical engine, force acts because of mass of piston,

$$W_R = m_R \cdot g$$

$$= 1.6 \times 9.81$$

$$W_R = 15.696 \text{ N}$$

Net thrust on the piston (or) Piston effort:

$$F_p = F_L - F_I + W_R$$

$$= 13253.6 - 5735.05 + 15.696$$

$$F_p = 7534.25 \text{ N}$$

Result:

Net thrust on piston, $F_p = 7534.25$ N.

5) A vertical single cylinder engine has a cylinder of 250 mm and stroke length of 450mm. The reciprocating parts have a mass of 180 kg. The connecting rod is 4 times the crank radius and the speed is 360 rpm. When the crank has turned through an angle at 45° from the top dead center, the net pressure on the piston is 1.05 MN/m². Calculate the effective turning moment on the crankshaft for this position.

Gn:

$D = 250 \text{ mm} = 0.25 \text{ m}$

$L = 450 \text{ mm} = 0.45 \text{ m}$

$\therefore r = \frac{0.45}{2} = 0.225 \text{ m}$

$P = 1.05 \text{ MN/m}^2 = 1.05 \times 10^6 \text{ N/m}^2$

$l = 4r = 4(0.225) = 0.9 \text{ m}$

$N = 360$ rpm

$\theta = 45°$

$m_R = 180$ kg

Soln:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(360)}{60} = 37.69 \text{ rad/sec}$$

$$n = \frac{l}{r} = \frac{0.9}{0.225} = 4$$

load on piston, $F_L = P \times A$

$$= 1.05 \times 10^6 \times \frac{\pi}{4}(0.25)^2$$

$$F_L = 51541.75 \text{ N}$$

Inertia force, $F_I = m_R \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$$= 180\,(37.69)^2\,(0.225)\left[\cos 45 + \frac{\cos 2(45)}{4}\right]$$

$$F_I = 40681.06 \text{ N}.$$

Weight of reciprocating part,

$$W_R = m_R \times g.$$

$$= 180 \times 9.81$$

$$W_R = 1765.8 \text{ N}$$

Piston effort, $F_P = F_L - F_I + W_R$.

$$= 51541.75 - 40681.06 + 1765.8$$

$$F_P = 12626.48 \text{ N}.$$

Turning moment on crank shaft,

$$T = \left[\frac{F_P}{\cos\phi}\sin(\theta+\phi)\right] r.$$

where, $\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 45}{4} = 0.177$

$$\therefore \phi = 10.18^\circ$$

$$\therefore T = \left[\frac{12626.48}{\cos 10.18}\sin(45+10.18)\right] 0.225$$

$$\boxed{\therefore T = 2219.588 \text{ N·m}}$$

Part - II / Turning Moment diagrams and flywheels:

Turning moment diagrams (or) Crank effort diagram:
X.2m

The turning moment diagram is the graphical representation of the turning moment for various positions of the crank (θ).

X.2m
Flywheel:

Flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

A flywheel serves as a mechanical reservoir for storing mechanical energy. It's function is to store the energy during the period when the supply of energy is more than the requirement and to give away the same when the requirement of energy is more than the supply.

Applications:

Flywheels are provided in engines and fabricating machines such as steel rollers, presses, shearing machines, riverting machines, punching machines, crushers, etc...

Flywheel Vs Governor:

Flywheel

Flywheel doesnot maintain a constant speed, it simply reduces the fluctuations speed. It doesnot control the speed of speed variations caused by varying load variations

Governor:

Governor regulates the speed of the engine when there are variations in the load.

Pbms:

[(11) b]:

1) In a turning moment diagram, the areas above and below the mean torque line taken in order 4400, 1150, 1300 and 4550 mm^2 respectively. The scales of turning moment diagram are

Turning moment, 1 mm = 100 N-m

Find the mass of the flywheel required to keep the speed between 297 and 303 r.p.m, if the radius of gyration is 0.525 mm.

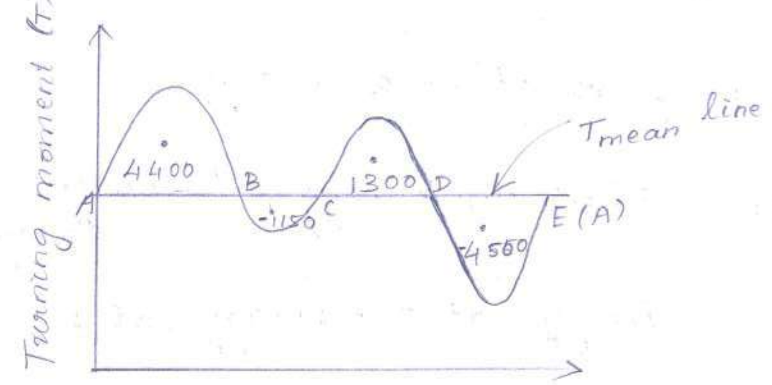
Gn:

$k = 0.525$ m.

$N_1 = 297$ rpm.

$N_2 = 303$ rpm.

Soln:



Crank angle (θ)   All areas are in mm^2

Given that, Turning moment, 1 mm = 100 N-m

Crank angle, 1 mm = 1°.

$1\,mm^2$ on turning moment diagram $= 100\left[1^\circ \times \frac{\pi}{180}\right]$

$= 1.745$ N.m

We find the energy of flywheel at various points on mean torque line as below.

Points	Energy (mm^2)	Remarks.
A	E	
B	E + 4400	
C	E+4400-1150 = E+3250	
D	E+3250+1300=E+4550	Maximum energy
E	E+4550-4550 = E	Minimum energy.

W.K.T,

Maximum fluctuation of energy,

ΔE = Maximum energy - Minimum energy.

= Energy at D - Energy at E.

$= (E + 4550) - E$

$\Delta E = 4550 \, mm^2$.

Considering scale value,

$\Delta E = 4550 \times 1.745$. ($1mm^2 = 1.745 \, Nm$)

$\Delta E = 7939.75 \, Nm$.

Mean speed of flywheel, $N = \frac{N_1 + N_2}{2}$

$= \frac{297 + 303}{2}$

$N = 300 \, rpm$.

$\therefore$ Mean angular velocity of flywheel,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(300)}{60} = 31.416 \, rad/s.$$

Coefficient of fluctuation of speed,

$$C_s = \frac{N_1 - N_2}{N}$$

$$= \frac{303 - 297}{300}$$

$C_s = 0.02$.

W.K.T, Maximum fluctuation of energy,

$\Delta E = I \omega^2 C_s$.

$I = m k^2$.

$\therefore \Delta E = m k^2 \omega^2 C_s$.

$7939.75 = m \, (0.525)^2 \, (31.416)^2 (0.02)$

$\therefore m = 1459.34 \, kg$.

2) The turning moment diagram for a petrol engine is drawn to a vertical scale of 1mm to 6Nm and horizontal scale of 1mm to 1°. The turning moment repeats itself after every half revolution of the engine. The areas above and

1mm = 6 Nm
1mm = 1°

below the mean torque line are 305, 710, 50, 350, 980 and 275 mm^2. Mass of reciprocating parts is 40 Kg at a radius of gyration of 140 mm. Calculate coefficient of fluctuation of speed, if the mean speed is 1500 rpm.

N

$C_s = 0.00548$.

0.548 %

$\Delta E : 106.05$ Nm.

3) A turning moment dig. for a multicylinder engine has been drawn to a scale of 1mm = 4500 Nm vertically and 1mm = 2.4° horizontally. The intersected areas between output torque curve and mean resistance line in order from one end are 342, 230, 245, 303, 115, 232, 227 and 164 mm^2, when the engine is running at 150 rpm (N). If the mass of the flywheel is 1000 kg and the ~~total~~ fluctuation of speed does not exceed 3% of mean speed, find the minimum value of radius of gyration.

$N_1 - N_2 = 3\% \cdot N \Rightarrow \frac{N_1 - N_2}{N} = 3\% \Rightarrow C_s = 3\%$

·x·x·
·x·

The turning moment diagram for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows.

Expansion stroke = 3550 mm^2.

Exhaust stroke = 500 mm^2.

Suction stroke = 350 mm^2.

Compression stroke = 1400 mm^2. $1\,mm^2 = 3\,Nm$

Each mm^2 represents $\underline{3\ Nm}$ scale value.

Assuming the resisting movement to be uniform, find the mass of the ring of a flywheel required to keep the mean speed 200 rpm, within $\pm 2\%$. The mean radius of the ring may be taken as 0.75 m. Also determine the crank positions for the maximum and minimum speeds.

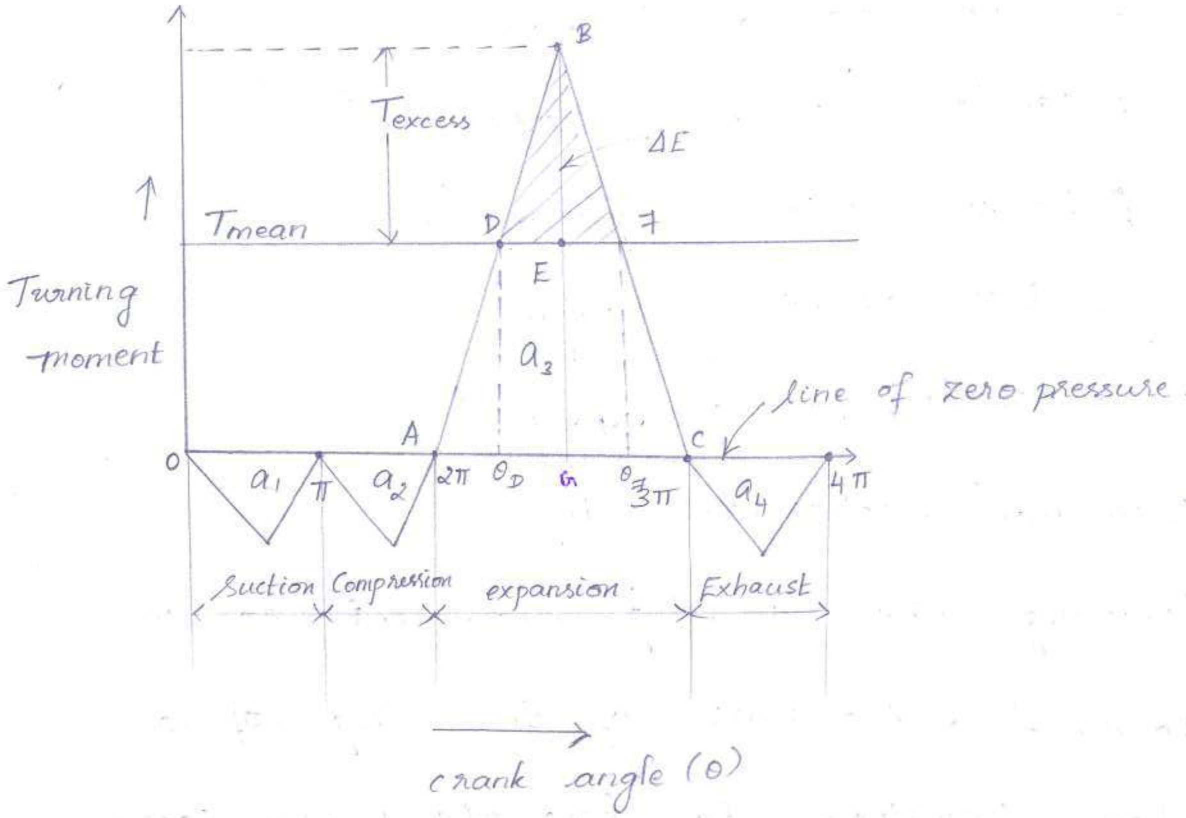
Sn:

$N = 200$ rpm.

$C_s = \pm 2\%$.

$R = k = 0.75$ m.

Soln:



$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi(200)}{60}$$

$\omega = 20.94$ rad/sec.

Workdone per cycle = Net area of the fig.

$= a_3 - (a_1 + a_2 + a_4)$

$= 3550 - (350 + 1400 + 500)$

$= 1300 \times 3$ (scale value).

$= 3900$ Nm.

For four stroke cycle,

Workdone per cycle $= T_{mean} \times 4\pi$ — ②.

$\therefore 3900 = T_{mean} \times 4\pi$

$\therefore T_{mean} = \frac{3900}{4\pi}$.

$T_{mean} = 310.35$ Nm.

The area above mean torque line represents the maximum fluxtuation of energy.

ΔE = Area of Δ^{le} BDF.

$= \frac{1}{2} DF \times BE$.

To find BE:

Workdone during expansion stroke,

a_3 = Area of Δ^{le} ABC.

$\Rightarrow 3550 \times 3 = \frac{1}{2} \times BG \times AC$.

$\therefore BG = 6780$ Nm.

Excess torque, $T_{excess} = BE$.

$= BG - EG$.
(T_{mean})

$= 6780 - 310.35$.

$= 6469.65$ Nm.

To find DF:

From similar triangles, DBF & ABC,

$$\frac{DF}{AC} = \frac{BE}{BG}.$$

$$\frac{DF}{\pi} = \frac{6469.65}{6780}$$

$\therefore DF = 2.9978$ N.m

To find ΔE:

ΔE = Area of ΔBDF.

$= \frac{1}{2} DF \times BE$

$= 9697$ N-m.

(i) Mass of rim of flywheel:

$$\Delta E = mk^2 \omega^2 C_s.$$

$$9697.32 = m(0.75)^2 (20.94)^2 (0.04).$$

$$m = 982.91 \text{ kg}.$$

(ii) Crank positions for maximum and minimum speed:

Let,

$\theta_D, \theta_F \rightarrow$ Crank angles from inner dead center for the minimum & maximum speeds

From similar triangles DBE and ABG

$$\frac{DE}{AG} = \frac{BE}{BG}.$$

$$\frac{DE}{\pi/2} = \frac{6469.65}{6780}$$

$\therefore DE = 1.4988$ ~~Nm~~ rad.

$\theta_D = AG - DE$.

$= \frac{\pi}{2} - 1.4988$

$\theta_D = 0.0718$ rad $= 0.0718 \times \frac{180}{\pi} = 4.12°$,

From similar triangles BEF and BGC,

$$\frac{EF}{GC} = \frac{BE}{BG}.$$

$$\frac{EF}{\pi/2} = \frac{6469.65}{6780}.$$

$EF = 1.4988$ rad.

$\theta_f = AG + EF$

$= \pi/2 + 1.4989$

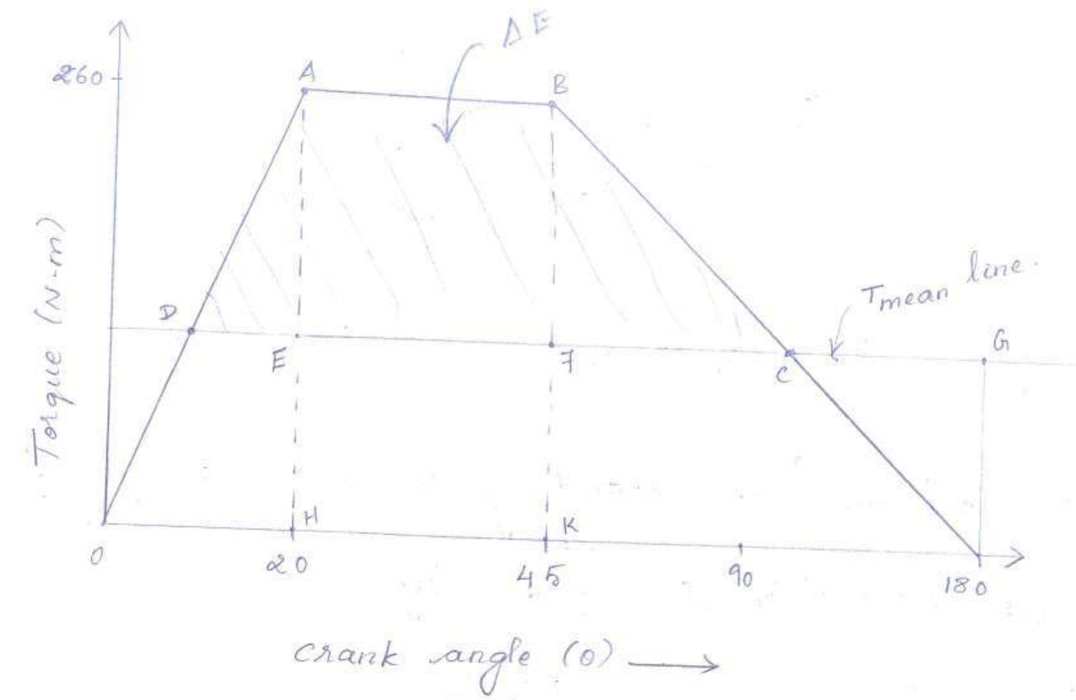
$\theta_f = 3.0692$ rad

$= 3.0692 \times \frac{180}{\pi}$

$\theta_f = 175.88°$.

The variation of crank shaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° and 180° and has 260 Nm for crank angles 20° and 45°, the intermediate portions of the torque graph being straight line. The cycle is being repeated in every half revolution. The average speed is 600 rpm. Supposing that the engine drives a machine requiring constant torque, determine the mass of flywheel of radius of gyration 250mm, which must be provided so that the total variation of speed shall be 1%.

Soln:



Gn:

$N = 600$ rpm.

$k = 250$ mm $= 0.25$ m

$C_s = 1\% = 0.01$.

Soln:

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi(600)}{60}$

$\omega = 62.83$ rad/s.

Workdone for half revolution = Area of turning moment diagram.

$= \begin{Bmatrix}\text{Area of} \\ OAH\end{Bmatrix} + \begin{Bmatrix}\text{Area of} \\ HABK\end{Bmatrix} + \begin{Bmatrix}\text{Area of} \\ KBG\end{Bmatrix}$.

$$= \left[\frac{1}{2} \left(\frac{20^\circ \times \pi}{180} \right) \times 260 \right] + \left[260 \times \left(\frac{(45^\circ - 20^\circ)\pi}{180} \right) \right] + \left[\frac{1}{2} \left(\frac{(180-45)\pi}{180} \right) \times 260 \right]$$

$= 465.13$ Nm.

Workdone corresponding of mean torque for half revolution is given by,

$$T_{mean} \times \pi = \text{Workdone}.$$

$$\therefore T_{mean} = \frac{465.13}{\pi}.$$

$$T_{mean} = 148.05 \text{ Nm}.$$

Since the area above the mean torque line represents maximum fluctuation of energy.

$\therefore$ The maximum fluctuation of energy,

ΔE = Area of DAE + Area of BFC + Area of EBAF

$$\Delta E = \left(\frac{1}{2} DE \times AE \right) + \left(\frac{1}{2} \times FC \times BF \right) + (EF \times AE). \quad \text{—(1)}$$

$$\frac{DE}{\left(\frac{20 \times \pi}{180} \right)} = \frac{(260 - 148.05)}{260}$$

$DE = 0.15$ rad.

From similar triangles, BGK and BFC,

$$\frac{FC}{KG} = \frac{BF}{BK}.$$

$$\frac{FC}{\frac{(180-45)\pi}{180}} = \frac{260 - 148.05}{260}$$

$\therefore FC = 1.014$ rad.

$$\therefore (1) \Rightarrow \Delta E = \left[\frac{1}{2} (0.15)(260 - 148.05) \right] + \left[\frac{1}{2} \times (1.014) \times (260 - 148.05) \right] + \left[\frac{(45-20)\pi}{180} \times (260 - 148.05) \right]$$

$= 8.4 + 56.75 + 48.84$.

$\Delta E = 114$ Nm.

W.K.T, $\Delta E = m k^2 \omega^2 C_s$.

$$114 = m(0.25)^2 (62.83)^2 (0.01).$$

$\therefore m = 46.21$ Kg.

The torque delivered by a two stroke engine is represented by $T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ Nm. where, 'θ' is the angle turned by the crank from the inner dead center. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine (i) The power developed (ii) The total percentage fluctuation of speed (iii) The angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner dead center (iv) the maximum angular acceleration and retardation of the flywheel.

Gn:

$T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ Nm.

$N = 250$ rpm.

$m = 400$ kg.

$K = 400$ mm.

Soln:

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi(250)}{60}$$

(i) Power developed by engine (P):

Since the given torque eqn. is a function of 2θ, the cycle of operation will be repeated after every π rad of the crank rotation.

if $\theta, 3\theta$

2π rad, $\frac{2\pi}{3}$ rad

$$\text{Workdone per cycle} = \int_0^{\pi} T \, d\theta$$

$$= \int_0^{\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta) d\theta$$

$$= \left[1000(\theta) + 300\left(\frac{-\cos 2\theta}{2}\right) - 500\left(\frac{\sin 2\theta}{2}\right)\right]_0^{\pi}$$

$$= \left[1000(\pi) + 300\left(\frac{-\cos 2\pi}{2}\right) - 500 \frac{\sin 2\pi}{2}\right] -$$

$$[0 - 150 - 0].$$

$$= 1000\pi \text{ N-m}.$$

$$\text{Mean resisting torque, } T_{mean} = \frac{\text{Workdone per cycle}}{\text{Crank angle per revolution}}$$

$$= \frac{1000\pi}{\pi}$$

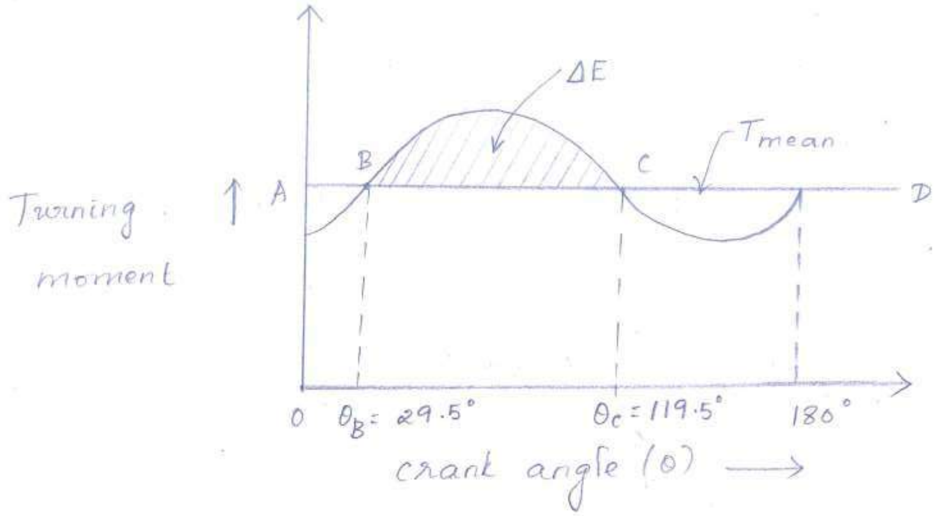
$$= 1000 \text{ N-m}.$$

$$\text{Power developed, } P = T_{mean} \times \omega = 1000 \times 26.18$$

$$= 26.18 \times 10^3 \text{ W}$$

(ii) Total percentage fluctuation of speed:

Turning Moment Diagram.



Here, the cycle of operation is repeated in every π revolution as shown in figure. The condition to find C_s on the flywheel is given by $T = T_{mean}$

$$1000 + 300 \sin 2\theta - 500 \cos 2\theta = 1000.$$

$\div \cos 2\theta$, $\Rightarrow 300 \tan 2\theta - 500 = 0$.

$$\tan 2\theta = \frac{500}{300} = \frac{5}{3} = 1.666.$$

$$\therefore 2\theta = 59.03$$

$$\Rightarrow 2\theta = 59^\circ.$$

Then, $\tan(180 + 2\theta) = \tan 2\theta$.

so, $\tan(180 + 59^\circ) = \tan 239^\circ$.

so, another value of 2θ is 239°

$$\therefore 2\theta = 59^\circ \; \& \; 239^\circ.$$

$\therefore$ Maximum fluctuation of energy, ΔE

$$\Delta E = \int_{29.5^\circ}^{119.5^\circ} (T - T_{mean})\, d\theta.$$

$$= \int_{29.5^\circ}^{119.5^\circ} \left[(1000 + 300 \sin 2\theta - 500 \cos 2\theta) - 1000\right] d\theta.$$

$$= \left[300\left(\frac{-\cos 2\theta}{2}\right) - 500\left(\frac{\sin 2\theta}{2}\right)\right]_{29.5^\circ}^{119.5^\circ}$$

$$= \left[300\left(-\frac{\cos 2(119.5^\circ)}{2}\right) - 500\left(\frac{\sin 2(119.5^\circ)}{2}\right)\right] - \left[300\left(-\frac{\cos 2(29.5^\circ)}{2}\right) - 500\left(\frac{\sin 2(29.5^\circ)}{2}\right)\right]$$

$$= (77.25 + 214.29) - (-77.25 - 214.29).$$

$$\Delta E = 583.08 \text{ Nm}.$$

$$\Delta E = m k^2 \omega^2 C_s.$$

$$583.08 = 400 \times (400 \times 10^{-3})^2 \times (26.18)^2 C_s.$$

$$\therefore C_s = 0.0133.$$

$$\therefore C_s = 1.33\%.$$

(iii) Angular accleration of flywheel when $\theta = 60°$:

At any moment,

$$I\alpha = (T - T_{mean}) \quad — ①.$$

$$I = mk^2.$$

$$mk^2\alpha = (1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000$$

$$(400)(0.4)^2 \alpha = 1000 + 300\sin 2(60) - 500\cos 2(60) - 1000.$$

$$\alpha = 7.965 \text{ rad/s}^2.$$

(iv) Maximum acceleration and retardation of flywheel:

here, $\frac{d}{d\theta} (T - T_{mean}) = 0$

$$\frac{d}{d\theta}\left[(1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000\right] = 0.$$

$$\left[2\times 300\left(\cos 2\theta\right) - 500\times 2\left(-\sin 2\theta\right)\right] = 0.$$

$\div \cos 2\theta$, $600 + 1000 \tan 2\theta = 0.$

$$1000 \tan 2\theta = -600.$$

$\therefore$ $2\theta = 30.96°$ (take +ve value)

then, $\tan(180 + 2\theta) = \tan 2\theta.$

$$\tan(180 + 30.96) = \tan 210.96.$$

$$\text{so, } 2\theta = 210.96°.$$

$\therefore$ The two values of 2θ are $30.96°$ & 210.96

Now for acceleration, add last angle value of ΔE

from figure,

$$\theta_c = 119.5°.$$

210.96
119.5
329.46

119.5
30.96
150.46

so, $2\theta = 149.04°$ & $329.04°$.

when $2\theta = 149.04°$, $T_1 = T - T_{mean}$.

$$T_1 = 1000 + 300\sin 2\theta - 500\cos 2\theta - 1000$$

$$= 300\sin 149.04° - 500\cos 149.04°.$$

$$\therefore T_1 = 583.1 \text{ Nm}.$$

when $2\theta = 329.04°$, $T_1 = T - T_{mean}$

$$= (1000 + 300\sin 2\theta - 500\cos 2\theta) - 1000$$

$$= 300\sin(329.04°) - 500\cos 329.04°$$

$$T_1 = -583.1 \text{ Nm}.$$

As values of $T - T_{mean}$ at maximum and minimum torque 'T' are same, the max. acceleration is eqn ① ⇒ $T - T_{mean} = I\alpha$.

$T - T_{mean} = mk^2\alpha = 583.1$

$mk^2\alpha = 583.1$.

$400\,(0.4)^2\,(\alpha) = 583.1$

$\therefore \alpha = 9.11\ rad/s^2$.

$\therefore$ Max. acceleration or retardation $= 9.11\ rad/s^2$.

Dimensions of flywheel rim:

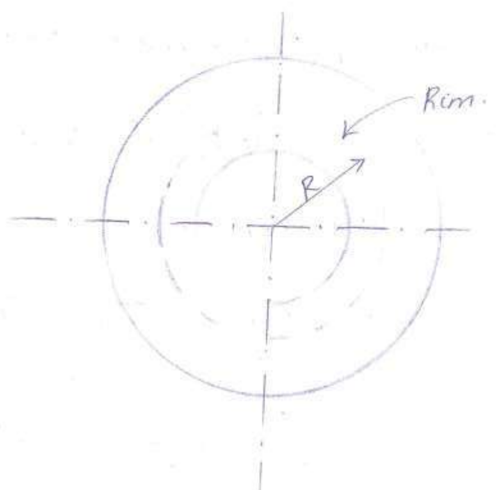


fig: flywheel rim

Let,

R → Mean radius of rim in 'm'

D → Mean diameter of rim in 'm'.

A → c/s of rim in m^2.

ρ → Density of rim material in kg/m^3.

N → Speed of the flywheel in rpm.

ω → Angular velocity of the flywheel in rad/sec

V → Linear velocity of the flywheel $= R\omega = \frac{\pi D N}{6}$

σ → Tensile or hoop stress due to centrifugal force in N/m^2.

Relation to be used are:

(i) Relation between α and v.

Hoop stress, $\sigma = \rho v^2$.

(ii) Relation for peripheral velocity (v)

$v = \sqrt{\sigma/\rho}$.

(iii) Mass of the rim, m = volume x density.

$m = \pi D A \times \rho$.

(iv) Area of rim (A):

$$A = \frac{m}{\pi D \rho}.$$

NOTE:

If c/s of the rim is rectangular, then

Area = width of rim (b) x Thickness of the rim (t).

1) A steam engine runs at 150 rpm. It's turning moment diagram gave the following area measurements in mm^2 taken in order above and below the mean torque line

500, −250, 270, −390, 190, −340, 270, −250,

The scale for turning moment is 1 mm = 500 Nm and the crank angle is 1 mm = 5°.

If the fluctuation speed is not to exceed ± 1.5% of the mean, determine the suitable diameter and c/s of the rim of the flywheel assumed with ~~actual~~ axial dimension = 1.5 times the radial dimension. The hoop stress is limitted to 3 MPa and the density of material of flywheel is 7500 kg/m^3.

Gn:

$N = 150$ rpm.

$\sigma = 3\ MPa = 3 \times 10^6\ N/m^2$.

$\rho = 7500\ kg/m^3$

$C_s = \pm 1.5\% = 3\% = 0.03$.

$b = 1.5 \times$ radial dimension.

$b = 1.5t$.

Soln:

$$\omega = \frac{2\pi N}{60}$$

$\omega = 15.71$ rad/s.

(i) Diameter of rim (D):

W.K.T, hoop stress, $\sigma = \rho v^2$.

$$3 \times 10^6 = 7500 \times v^2.$$

$v = 20$ m/s.

$$v = \frac{\pi D N}{60}$$

$$\Rightarrow 20 = \frac{\pi D (150)}{60}$$

$\therefore D = 2.546$ m.

(ii) c/s of flywheel rim (b & t):

Let us find the ΔE,

Turning momoment scale : 1 mm = 500 Nm.

Crank angle : 1 mm = 5°.

so, $1\ mm^2$ on the turning moment diagram

$$= 500 \times \left(5° \times \frac{\pi}{180}\right)$$

$= 43.63$ Nm.

Point	Energy (mm^2)	Remarks
A	E	
B	E +500	
C	E +500 - 250 = E + 250	
D	E + 250 + 270 = E + 520	Maximum Energy
E	E +520 - 390 = E + 130	
F	E + 130 + 190 = E + 320	
G	E + 320 - 340 = E - 20	Minimum energy.
H	E - 20 + 270 = E + 250.	
I	E + 250 - 250 = E = Energy at 'A'.	

We know that maximum fluctuation of energy ΔE = Maximum energy - Minimum energy.

= Energy at 'D' - Energy at 'G'.

$= (E + 520) - (E - 20)$

$= 540 \ mm^2$

= 540 x scale value.

$= 540 \times 43.63$

$\Delta E = 23560.2 \ Nm.$

K → mean radius (R)

We know that, $\Delta E = mk^2\omega^2 C_s = mR^2\omega^2 C_s$

$23560.2 = mv^2 C_s \quad [\because v = R\omega]$

$23560.2 = m(20^2)(0.03)$

We know that, mass of flywheel rim,

m = volume x Density.

$m = (\pi \times D \times A) \times \rho.$

$1963.35 = \pi (2.546) A (7200).$

$\therefore A = 0.0341 \ m^2.$

here, A = width x thickness = bt.

$\Rightarrow 0.0341 = 1.5t \times t.$

$0.0341 = 1.5t^2.$

$\Rightarrow t = 0.1507 \ m.$

$t = 150.7 \ mm.$

$\therefore$ Width of the rim, $b = 1.5t.$

$= 1.5(0.1507)$

$b = 0.226 \ m.$

$b = 226 \ mm.$

Result:

(i) Diameter of the rim = 2.546 m.

(ii) Cross section of the flywheel rim:

$b = 0.226 \ m$

$t = 0.1507 \ m.$

BALANCING

Balancing of Rotating masses:

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses.

PROBLEMS ON BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE:

Pbm 1: Four masses m_1, m_2, m_3, m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required. If its radius of rotation is 0.2 m.

GIVEN DATA:

$m_1 = 200$ kg

$m_2 = 300$ kg

$m_3 = 240$ kg

$m_4 = 260$ kg

$r_1 = 0.2$ m

$r_2 = 0.15$ m

$r_3 = 0.25$ m

$r_4 = 0.3$ m

$\theta_1 = 0^\circ$

$\theta_2 = 45^\circ$

$\theta_3 = 45^\circ + 75^\circ = 120^\circ$

$\theta_4 = 45^\circ + 75^\circ + 135^\circ = 255^\circ$

$r = 0.2\ m$

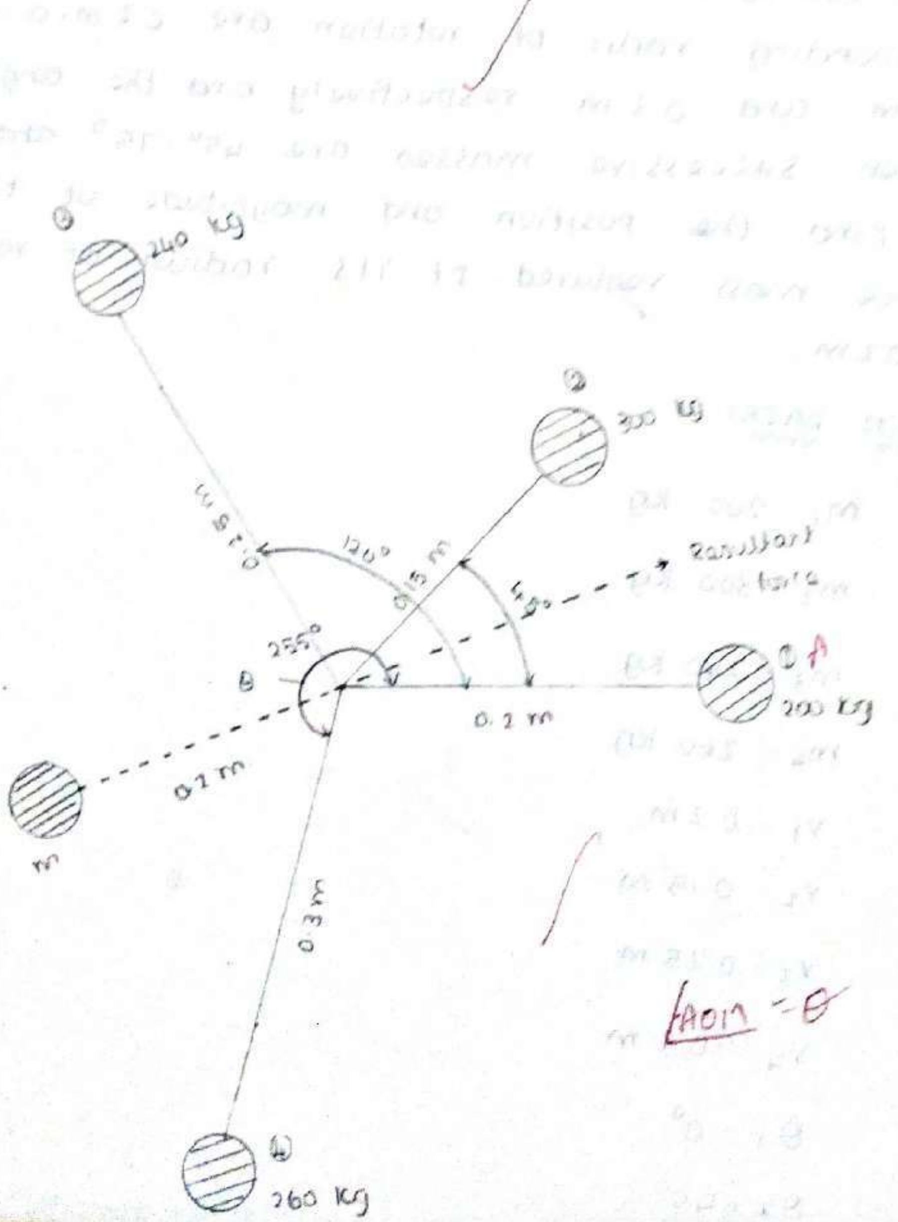
Let m = balancing mass.

θ = the angle between which the balancing mass makes with m_1

Graphical method:

The magnitude and the positions of the balancing mass may be found graphically as discussed below.

Space diagram:



since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

$m_1 \cdot r_1 = 200 \times 0.2 = 40$ kg-m

$m_2 \cdot r_2 = 300 \times 0.15 = 45$ kg-m

$m_3 \cdot r_3 = 240 \times 0.25 = 60$ kg-m

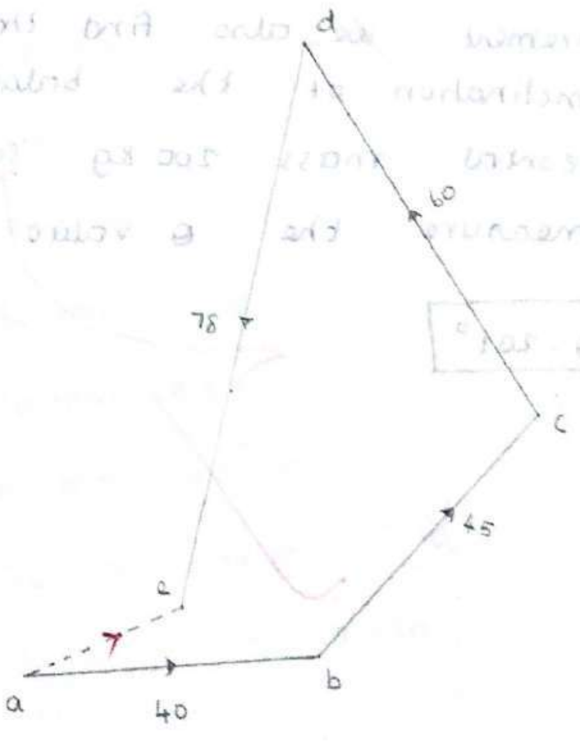
$m_4 \cdot r_4 = 260 \times 0.3 = 78$ kg-m.

Now draw the vector diagram with a above values to some suitable scale as shown in figure.

vector diagram:

scale:

1 cm : 10 kg m.



The closing side of the polygon ao represents the resultant force. 2.3

Scale

$= 23$ kg-m $\quad 2.3 \times 10 = 23$

The balancing force is equal to the resultant force shown in figure A.

Since the balancing force is proportional

W.K.T.

$m.r$

Given $r = 0.2$

$m \times 0.2 = 23$ kgm

$\therefore m = \frac{23}{0.2}$

$m = 115$ kg

By measurement we also find that the angle of inclination of the balancing mass (m) from horizontal mass 200 kg counter clockwise direction. measure the θ value

$\boxed{\theta = 201^\circ}$

problems on balancing mass of several masses rotating in different planes:

Pbm 1. A shaft carrys four masses A, B, C, D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anti clockwise are A to B 45°, B to C 70°, C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm and between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses resolved at a radius of 100 mm, find their magnitude and angular positions.

Given data:

$m_A = 200$ kg

$m_B = 300$ kg

$m_C = 400$ kg

$m_D = 200$ kg

$r_A = 80$ mm $= 0.08$ m

$r_B = 70$ mm $= 0.07$ m

$r_C = 60$ mm $= 0.06$ m

$r_D = 80$ mm $= 0.08$ m

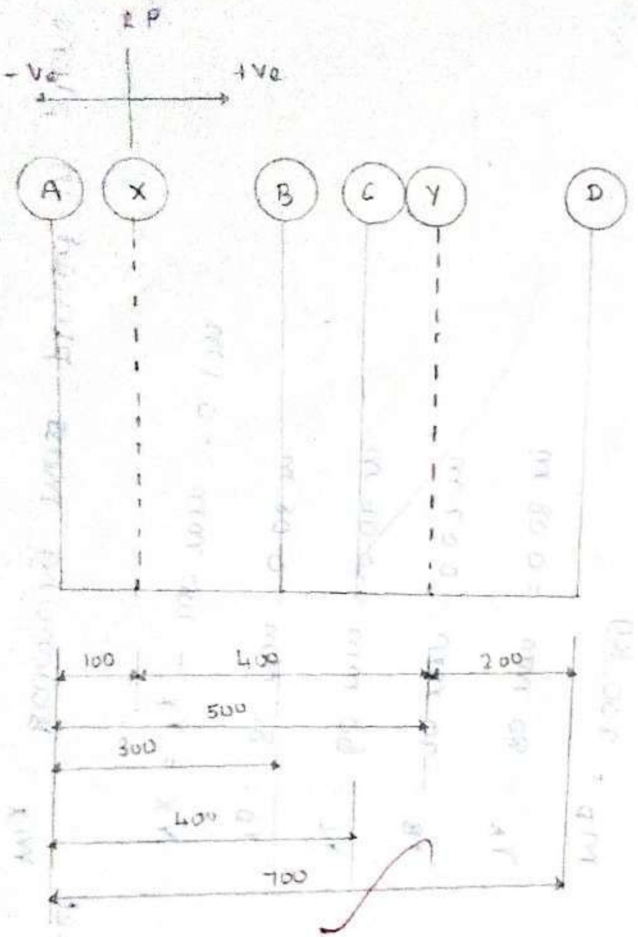
$r_X = r_Y = 100$ mm $= 0.1$ m

Let

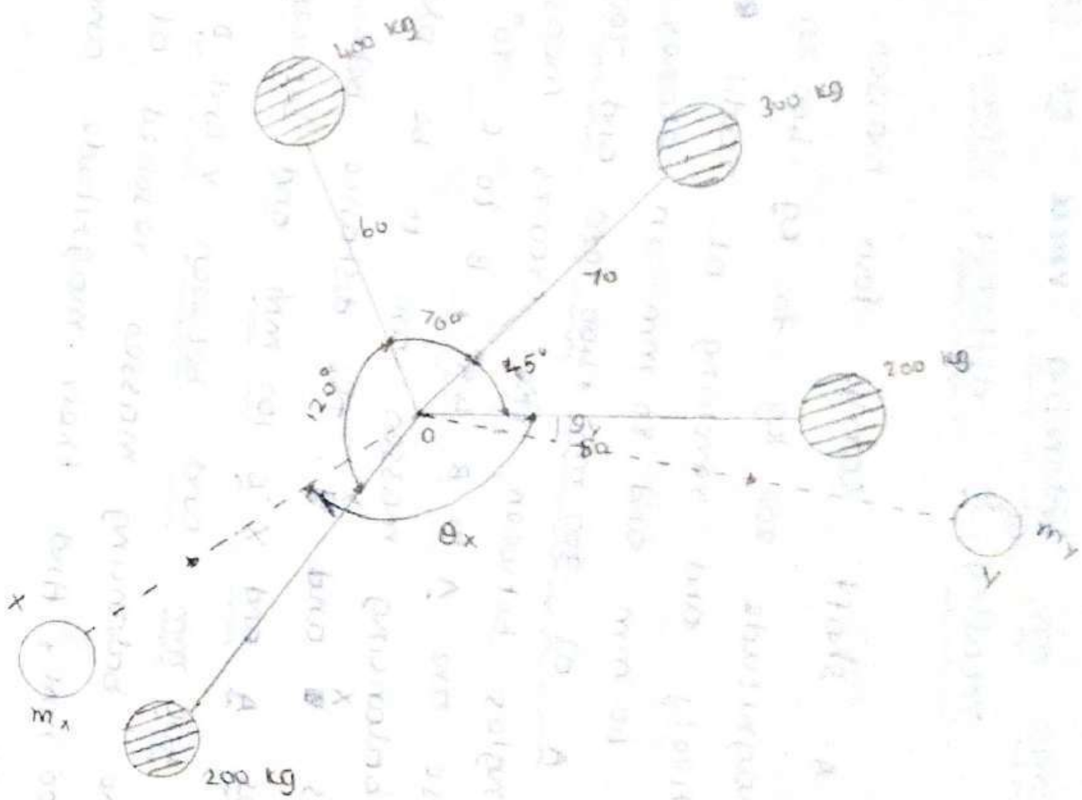
m_X = Balancing mass placed in plane X, and

m_Y = Balancing mass placed in plane Y.

a, position of planes:



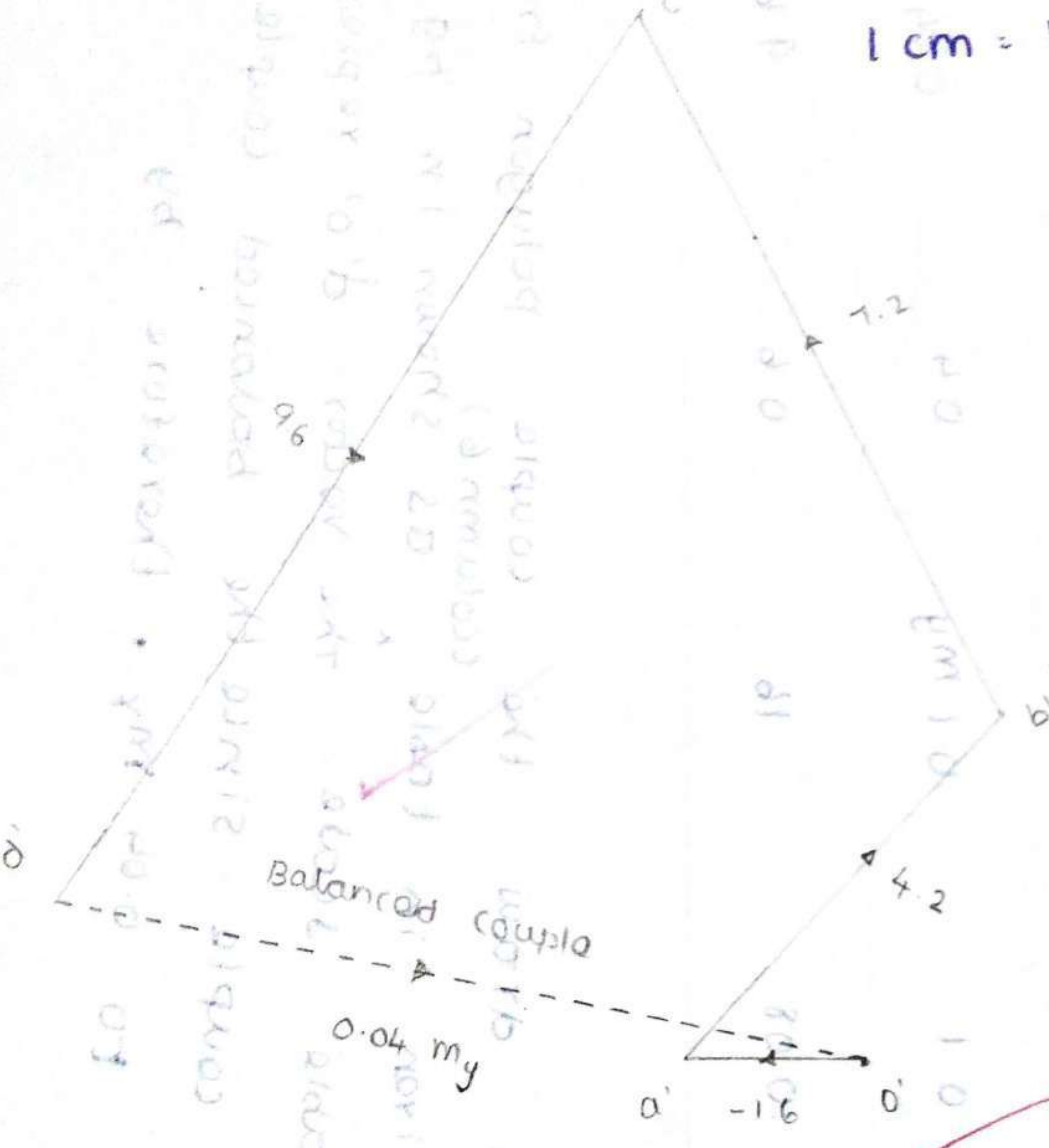
b. Angular position of masses



c. couple polygon:

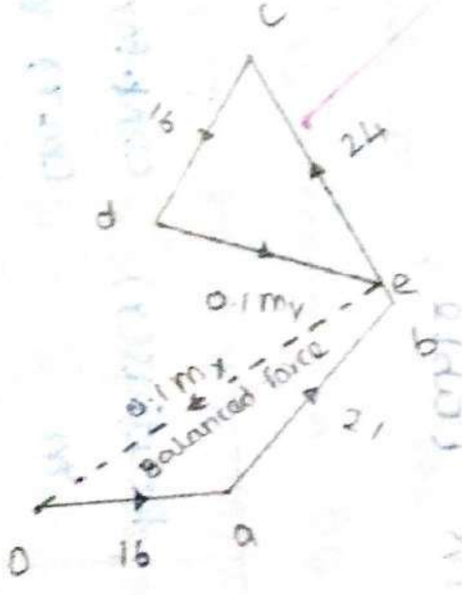
Scale:

1 cm = 1 kg·m²



d. Force polygon :

scale : 1cm = 10 kg-m.



a. position of planes

the position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in fig (a) and (b) respectively.

Assume the plane X as the reference plane (R.P). the distances of the planes to the right of plane X are taken as +ve while the distances of the planes to the left of plane X are taken as -ve. the data may be tabulated as shown in table.

plane	Mass(m) kg	Radius(r) m	cent. force $\div \omega^2$ (m.r) kg.m	Distance from Ref. plane x(l) m	couple $\div \omega^2$ (m.r.l) (kg-m²)
A	200	0.08	16	-0.1	-1.6
X (R.P)	m_x	0.1	$0.1\,m_x$	0	0
B	300	0.07	~~21~~ 21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1\,m_y$	0.4	$0.04\,m_y$
D	200	0.08	16	0.6	9.6

1. First of all draw the couple polygon from the data given in table (column 6) as shown in fig. to some suitable scale. The vector $d'o'$ represents the balanced couple. since the balanced couple is proportional to $0.04\,m_y$, therefore by measurement.

$$0.04\,m_y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$$

$\boxed{m_y = 182.5 \text{ kg}}$

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. parallel to vector d'o'. By measurement, the angular position of m_Y is $\boxed{\theta_Y = 12^\circ}$ in the clockwise direction from mass m_A (i.e 200 kg).

$$\boxed{\theta_Y = 12^\circ}$$

2. Now draw the polygon from the data given in table (column 4) as shown in fig (a). The vector eo represents the balanced force. since the balanced force is proportional to $0.1\, m_X$, therefore by measurement

scale
$3.45 \times 10 = 34.5$

$$0.1\, m_X = \text{vector } eo = 34.5 \text{ kg-m}$$

$$\boxed{m_X = 345 \text{ kg}}$$

The angular position of mass m_X is obtained by drawing Om_X in Fig (b), parallel to vector eo. By measurement, the angular position of m_X is $\theta_X = 145^\circ$ in the clockwise direction from mass m_A (i.e 200 kg).

$$\boxed{\theta_X = 145^\circ}$$

3. Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	—	30	50	40
Radius (mm)	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find

1. The magnitude and the angular position of mass A; and
2. The position of planes A and D.

Given data

$r_A = 180\ mm = 0.18\ m$

$m_B = 30\ kg$

$r_B = 240\ mm = 0.24\ m$

$m_C = 50\ kg$

$r_C = 120\ mm = 0.12\ m$

$m_D = 40\ kg$

$r_D = 150\ mm = 0.15\ m$

$\angle BOC = 90°$

$\angle BOD = 210°$

$\angle COD = 120°$

Soln.

1. The magnitude and the angular position of mass A.

Let

m_A = Magnitude of mass A.

x = Distance between the planes B and D,

y = Distance between the planes A and B,

The position of the planes and the angular position of the masses is shown in figure (a) & (b) respectively.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg m (4)	Distance from Reference plane (B) (l) m (5)	Couple ÷ ω^2 (m.r.l) kg. m^2 (6)
A	m_A	0.18	$0.18\ m_A$	$-y$	$-0.18\ m_A y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$

1, the closing side (vector do) is proportional to $0.18\ m_A$. By measurement,

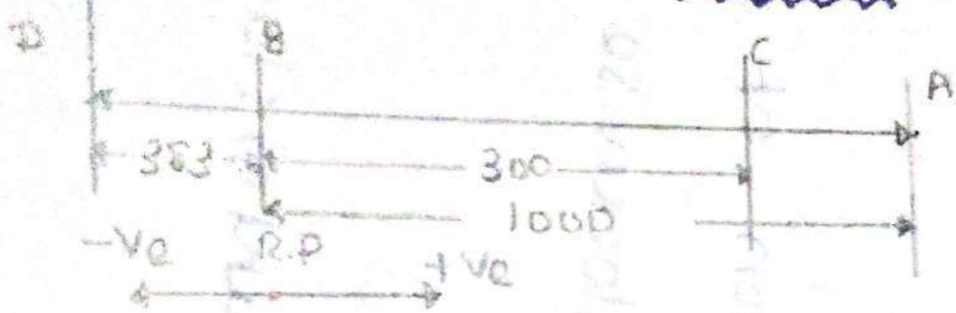
$0.18\ m_A$ = vector do = 3.6 kg-m

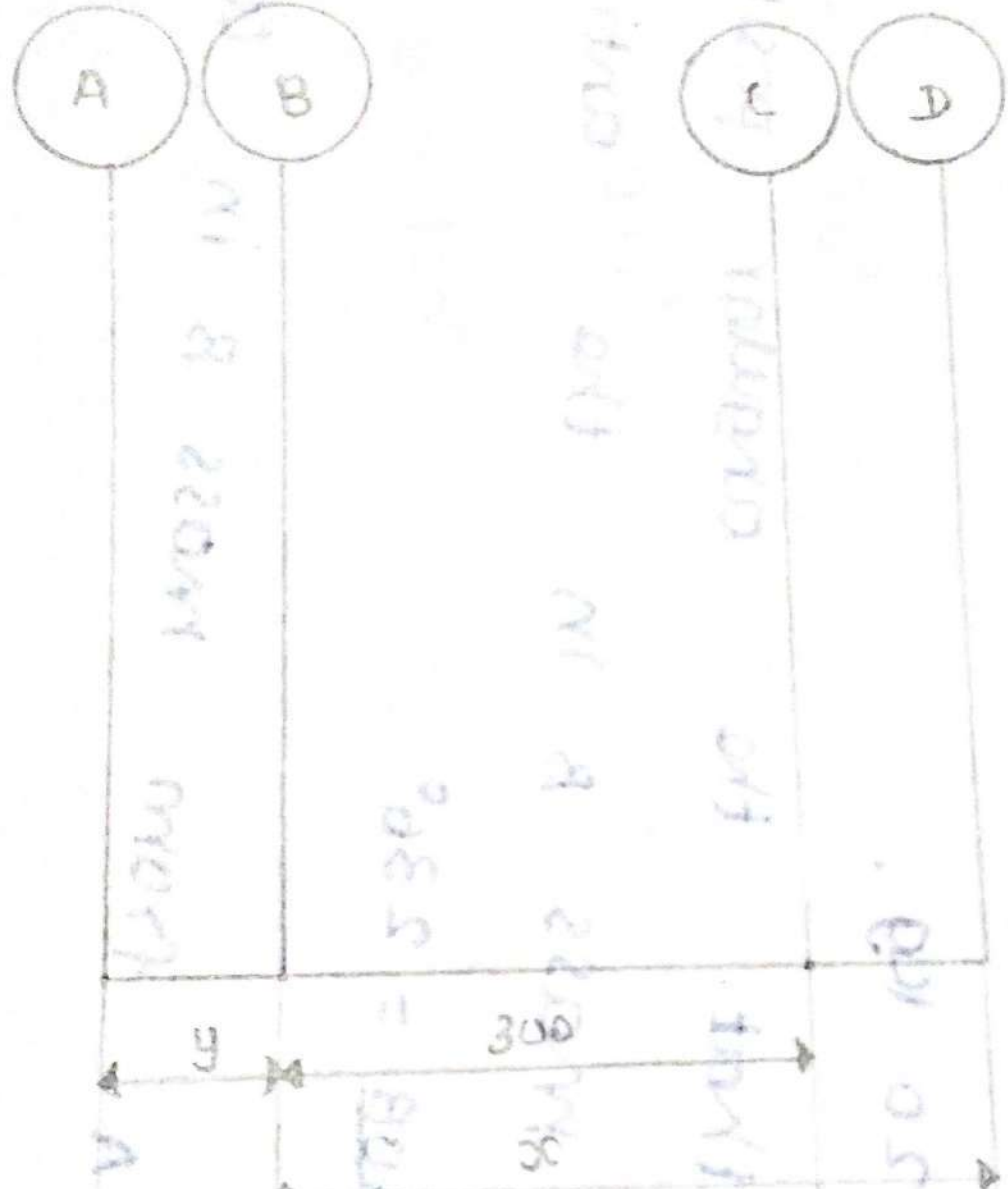
m_A = 20 kg.

We find that the angular position of mass A from mass B in the anticlockwise direction is $\angle AOB = 236°$.

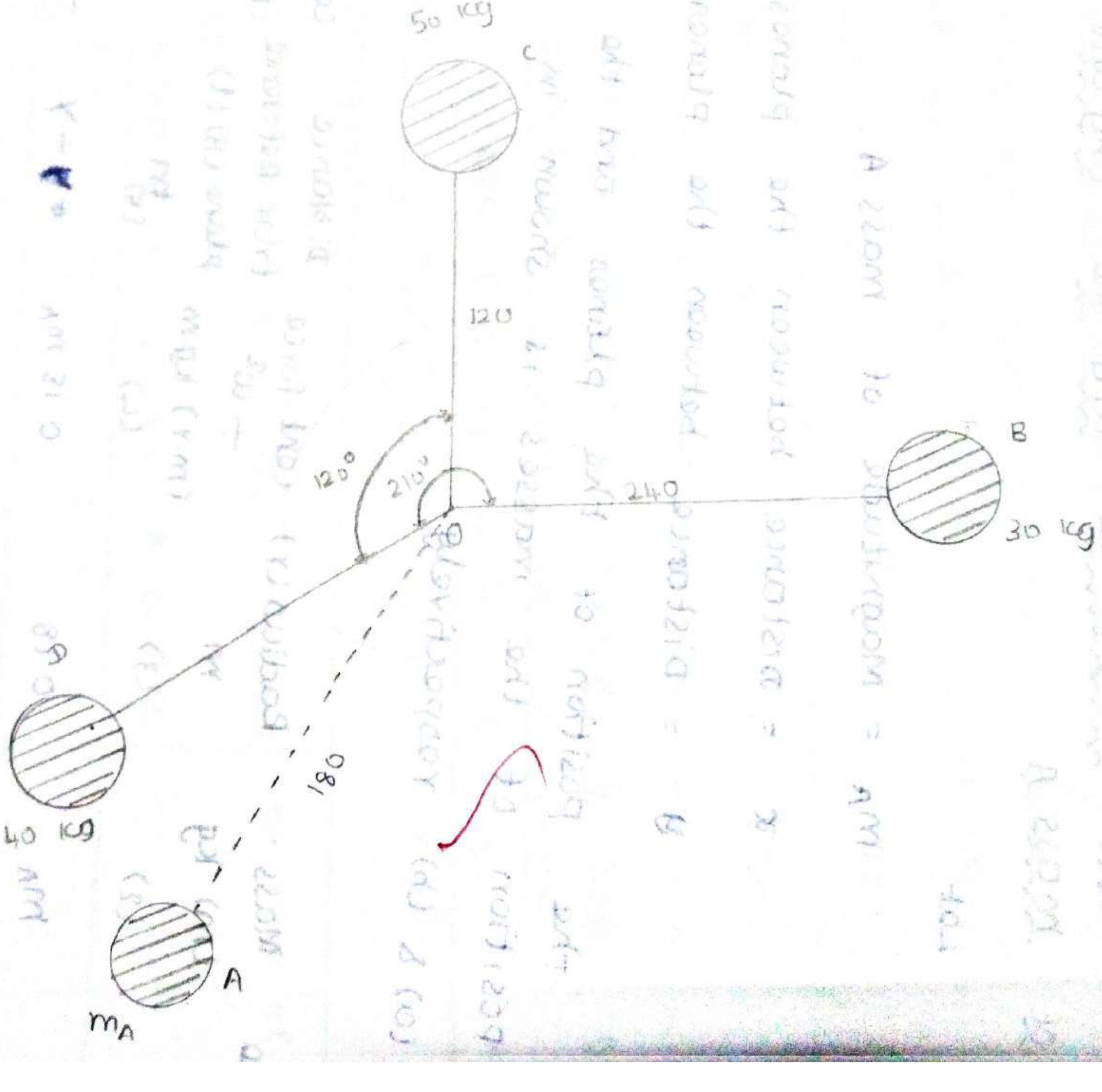
$\angle AOB = 236°$ A from mass B in counterclockwise direction.

a. position of planes



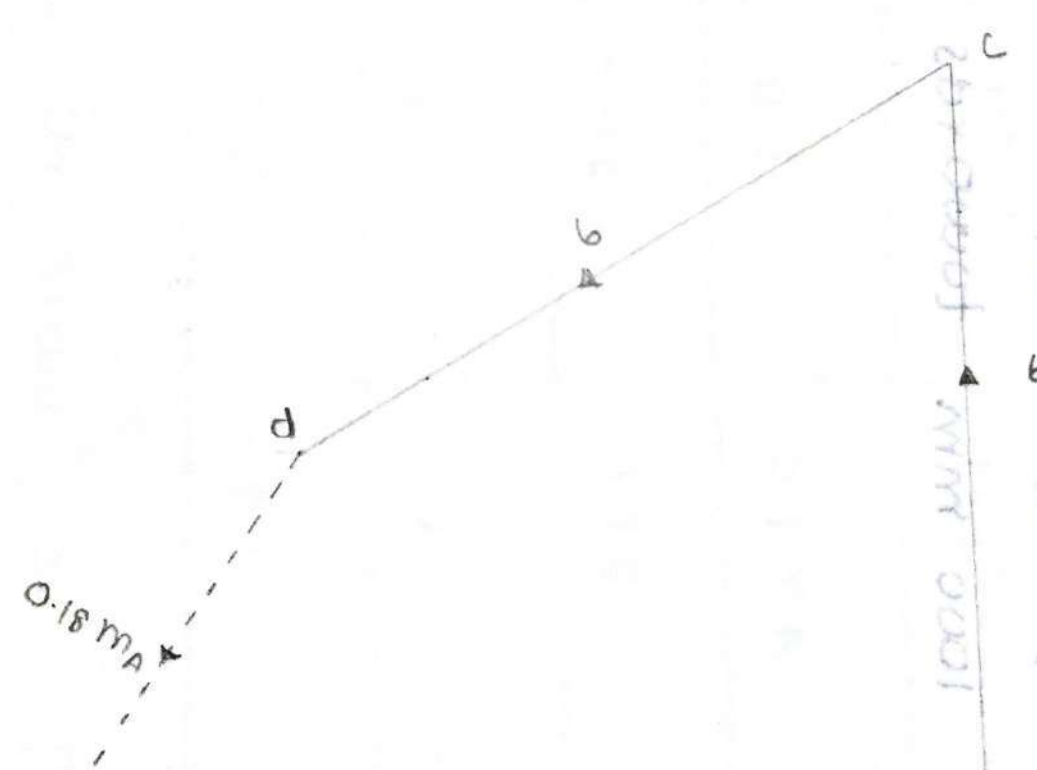


b. Angular position of masses:



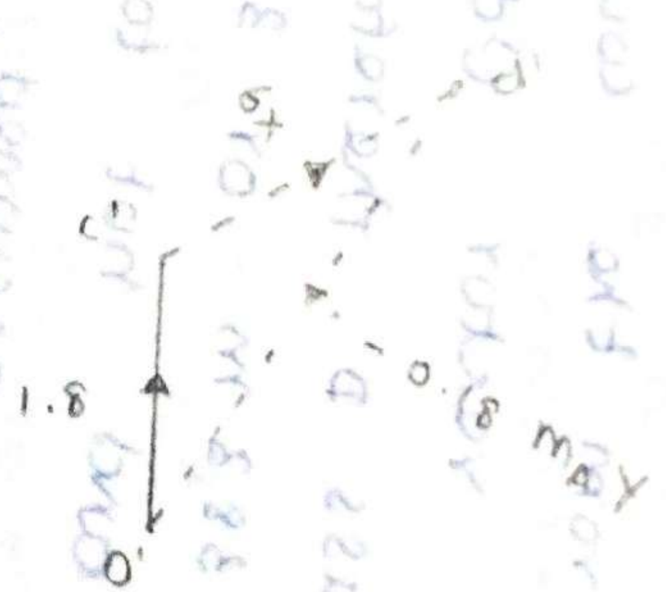
c. Force of polygon.

scale 1cm : 1kg.m



d. couple polygon

Scale: 1cm = 1kg.m^2



From points c' and o', draw lines parallel to OD and OA respectively, such that they intersect at point d'. By measurement, we find that

$$6x = \text{vector } c'd' = 2.3 \text{ kg-m}^2$$

$$x = 0.383 \text{ m}.$$

We see from the couple polygon that the direction of vector c'd' is opposite to the direction of mass D. Therefore the plane of mass D is 0.383 m or 383 mm towards left of plane B and not towards right of plane B as already assumed.

From that couple polygon. were o'd'

By measurement

$$-0.18\, m_A . y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-0.18 \times 20\, y = 3.6 \quad \text{(or)} \quad y = -1 \text{ m}$$

The negative sign indicates that the plane A is not towards left of B assumed but it is. 1 m or 1000 mm towards right of plane B

Pbm 4. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the masses of B, C and D are 10 kg, 5 kg and 4 kg respectively.

Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Given data:

$r_A = 100$ mm $= 0.1$ m $r_B = 125$ mm $= 0.125$ m

$r_C = 200$ mm $= 0.2$ m $r_D = 150$ mm $= 0.15$ m

$m_B = 10$ kg ; $m_C = 5$ kg

$m_D = 4$ kg.

Soln:

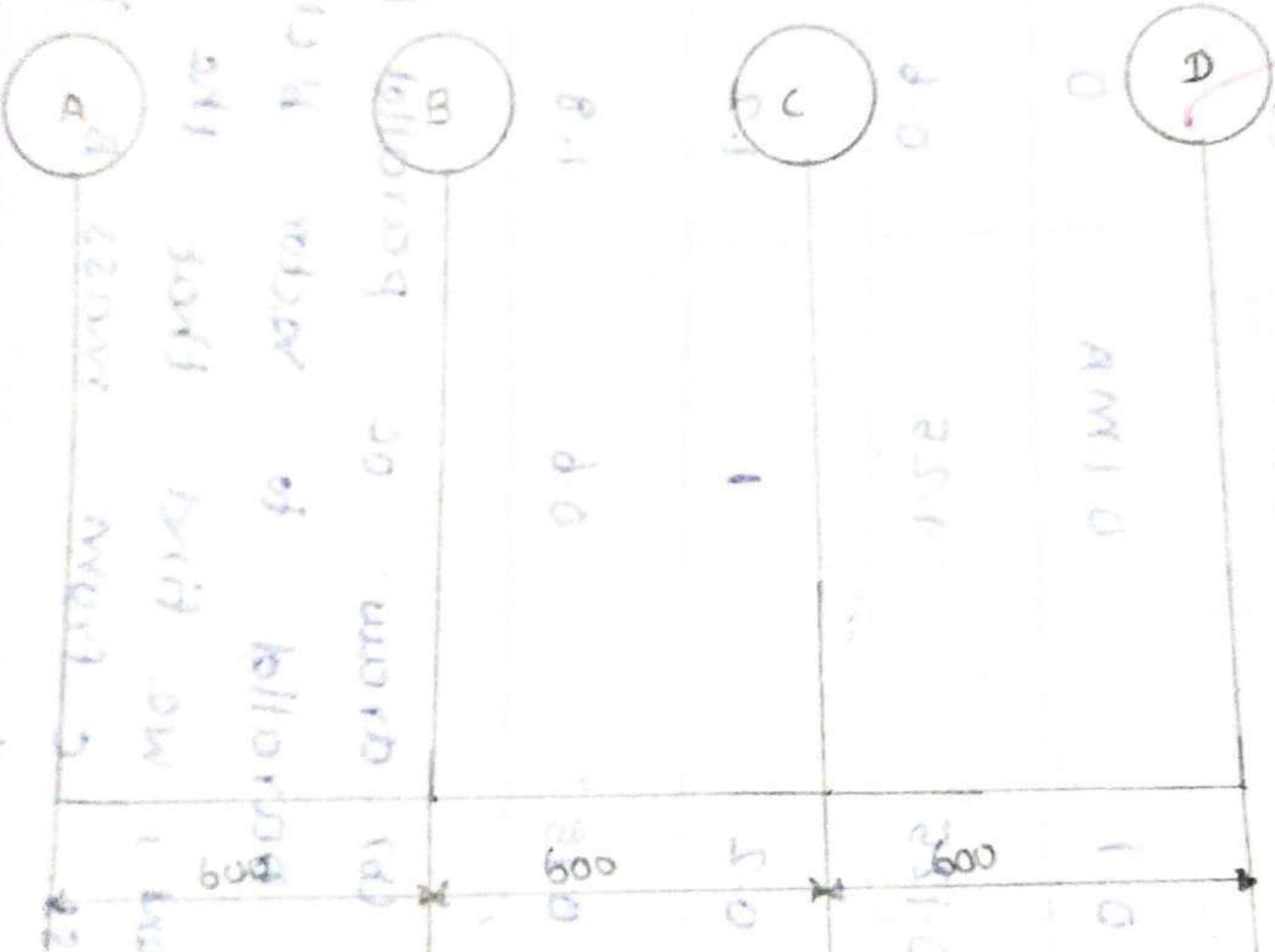
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force $\div \omega^2$ (m.r) kg.m (4)	Distance from Ref. plane A (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg.m^2 (6)
A (R.P)	m_A	0.1	$0.1 m_A$	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

Now figure (b) draw OC parallel to vector $o'c'$ and OD parallel to vector $b'c'$.

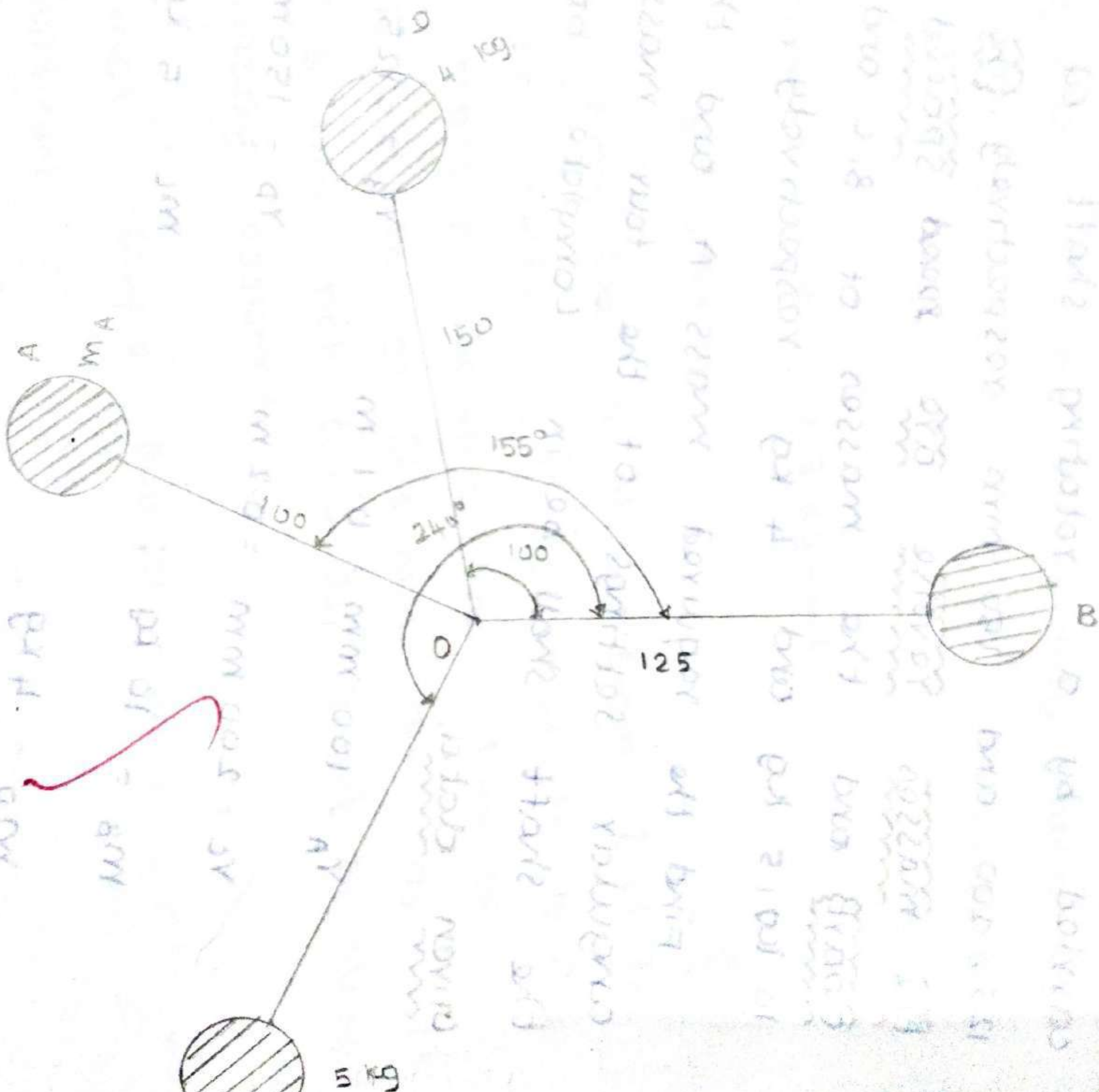
By measurement, we find that the angular setting of mass C from mass B in the anticlockwise direction.

$\angle BOC = 240°$.

a. position of planes

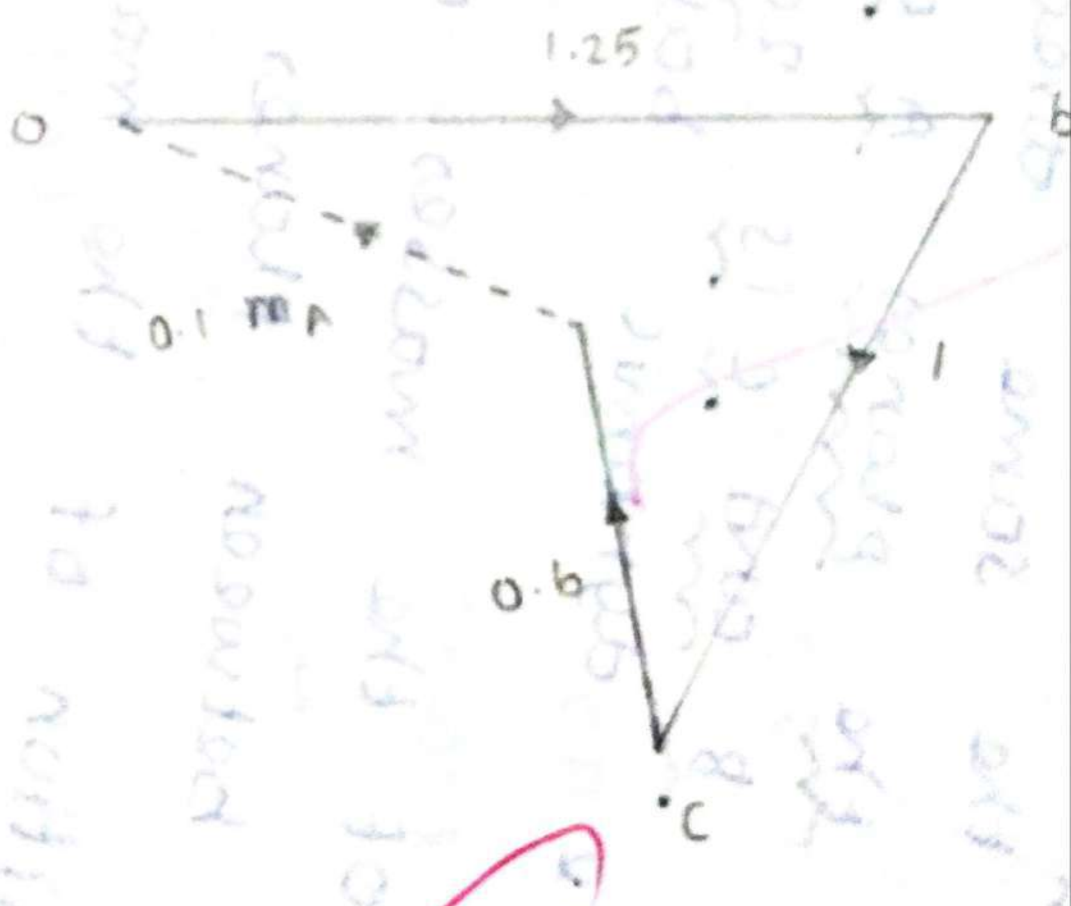


b. Angular position of masses



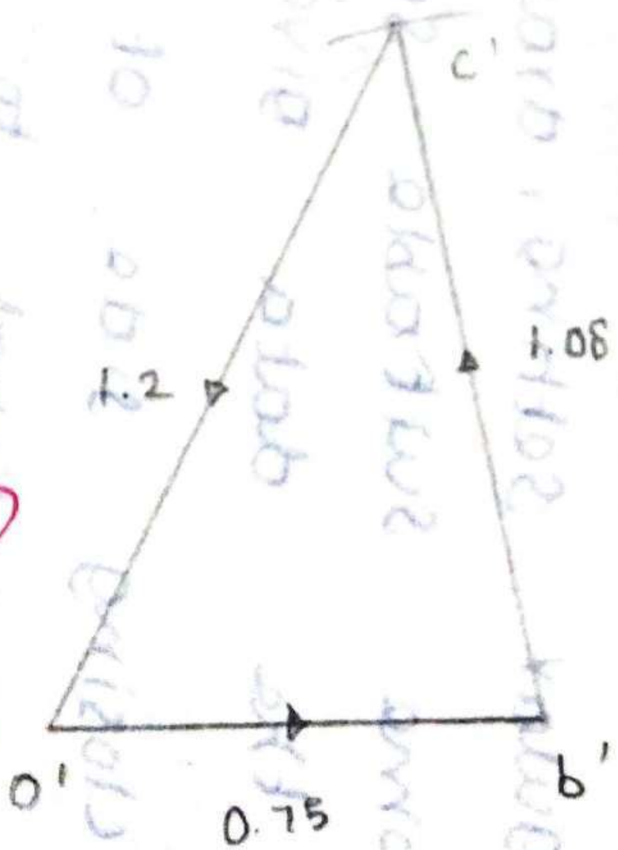
d. Force Polygon.

Scale: 1cm = 0.25 kg-m.



c d. couple polygon:

scale: $1\,cm = 0.25\ kg\ m^2$



and angular setting of mass D from mass B in the anticlockwise direction.

$\angle BOD = 100^\circ$

In order to find the required mass A (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in fig (d), from the data given in table (column 4)

Since the closing side of the force polygon (vector do) is proportional to $0.1\ m_A$, therefore by measurement.

scale

$0.1\ m_A = 2.8 \times 0.25$

$0.1\ m_A = 0.7\ kg\text{-}m^2$

$m_A = 7\ kg.$

5. A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine

1. The magnitude of the masses at A and D
2. The distance between planes A and D; and
3. The angular position of the mass at D.

Given data

$m_B = 18$ kg

$m_c = 12.5$ kg

$r_B = r_C = 60$ mm $= 0.06$ m

$r_A = r_D = 80$ mm $= 0.08$ m

$\angle BOC = 100°$

$\angle BOA = 190°$

Soln.

plane (1)	Mass (m) kg (2)	Eccentricity (r) m (3)	Cent-force $\div \omega^2$ (m.r) kg-m (4)	Distance from Ref. plane A (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg.m² (6)
A (R.P)	m_A	0.08	$0.08\ m_A$	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	$0.08\ m_D$	x	$0.08\ m_D . x$

1, By measurement.

we find that

($7.8 \times 0.03 = 0.234$

$0.08\ m_D\ x$ = vector $c'o'$ = 0.234 kg-m²

In figure (b) draw OD parallel to vector $c'o'$

to fix in the direction of mass D.

(scale $2.5 \times 0.3 = 0.75$

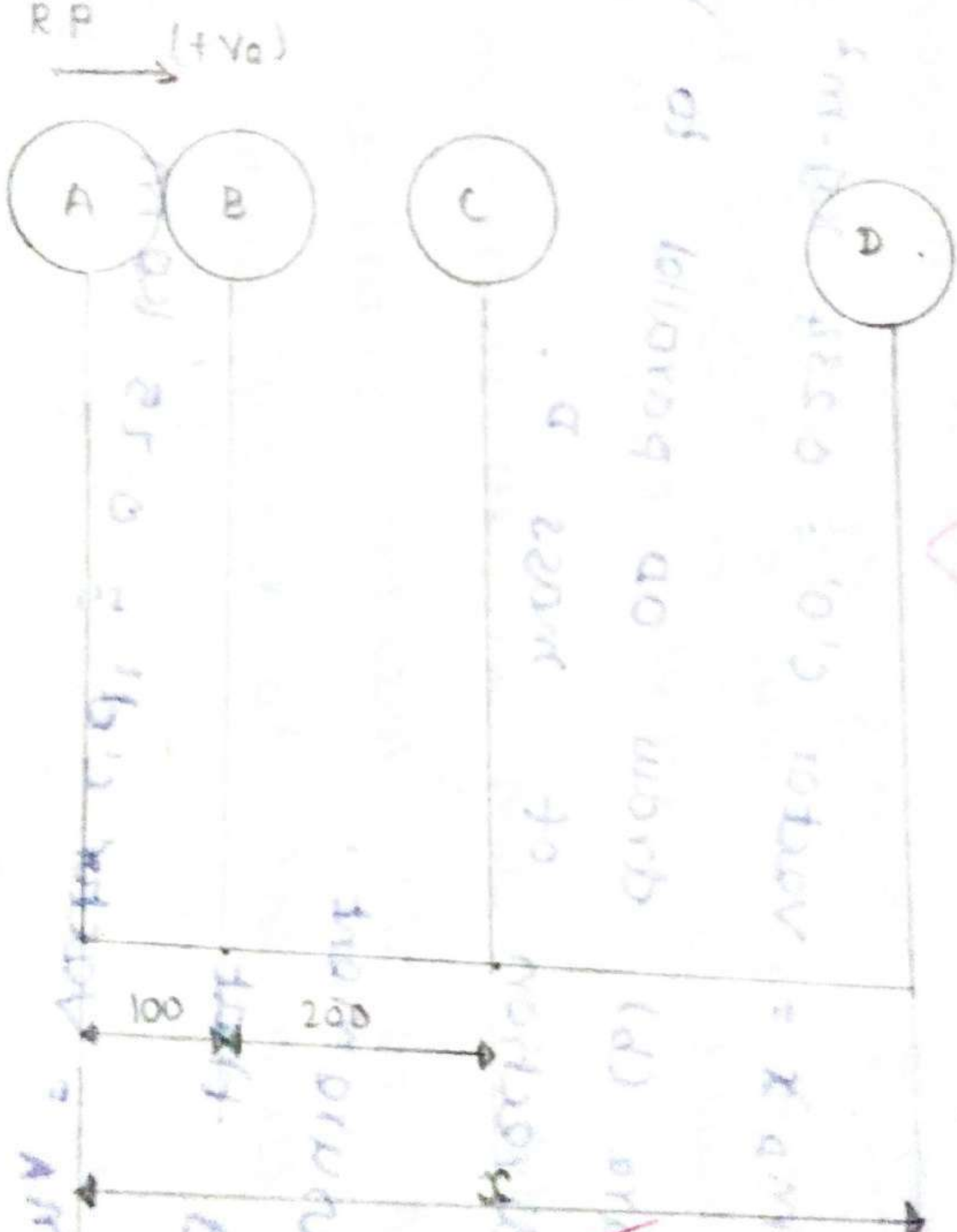
2. By measurement.

we find that

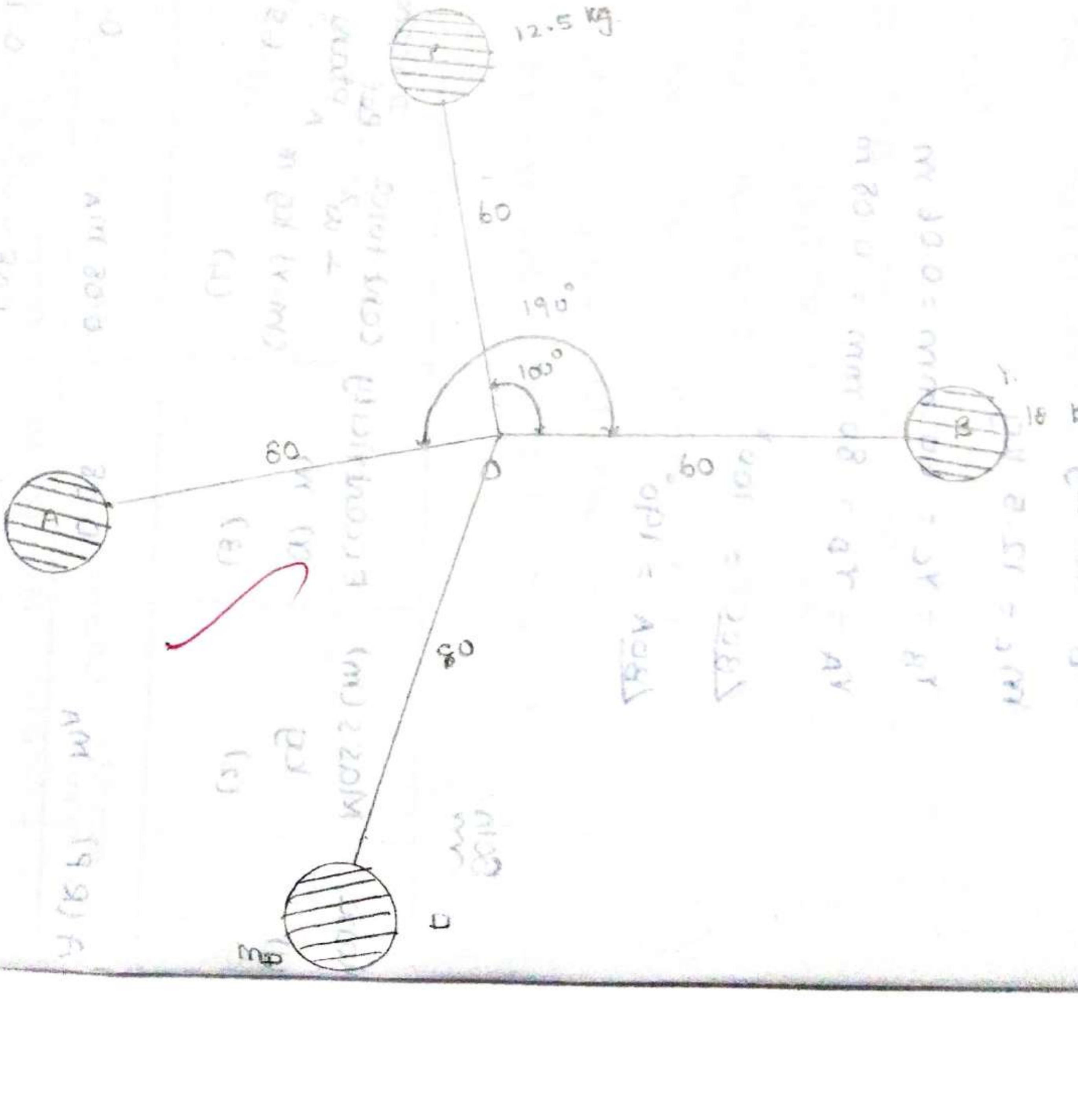
$0.08\ m_A$ = vector $c'd'$ = 0.75 kg.m

$\boxed{\therefore m_A = 9.375 \text{ kg}}$

a. position of planes:



b. Angular position of masses.



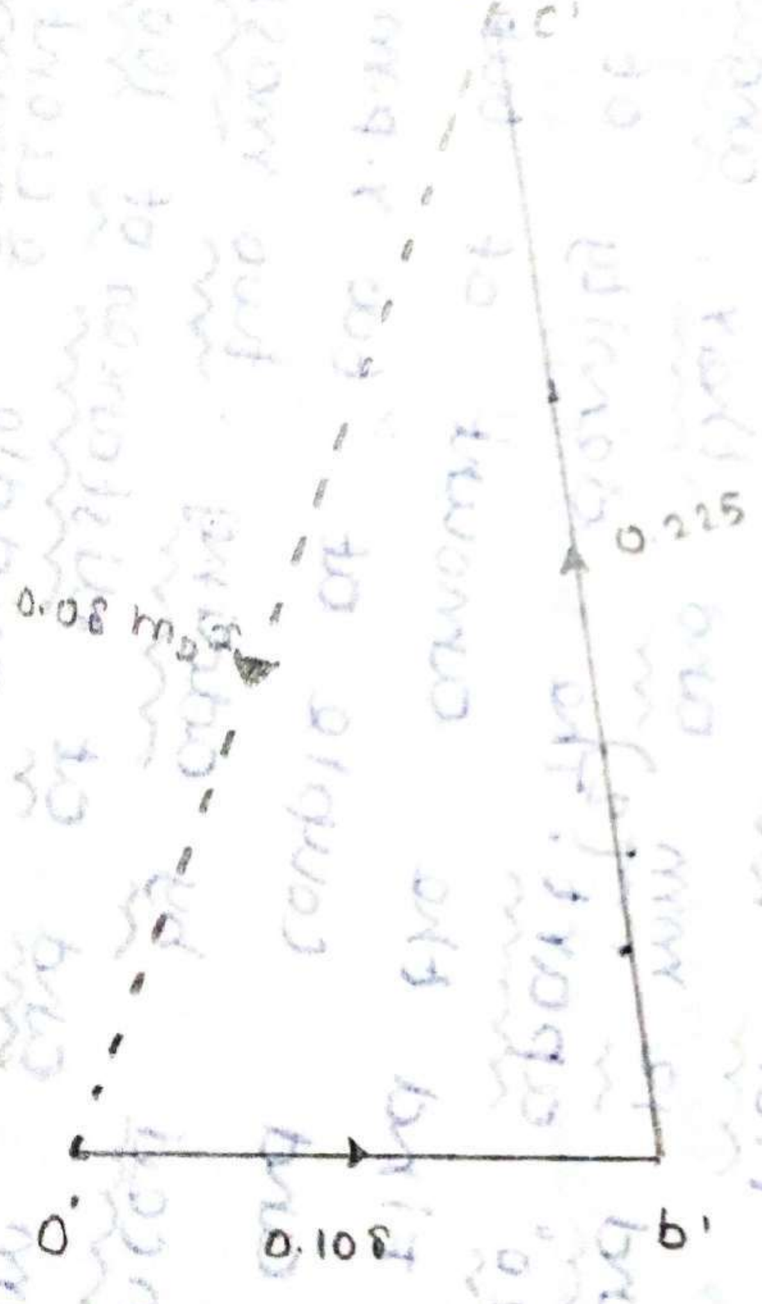
c, couple polygon :

scale:

1 cm : 0.03 kg m2

$\frac{0.108}{0.03} = 3.6\text{ cm}$

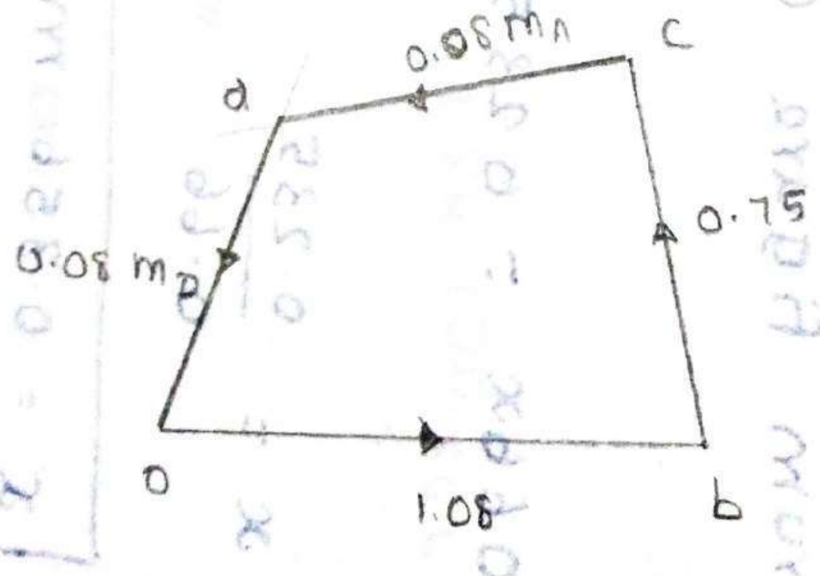
$\frac{0.225}{0.03} = 7.5\text{ cm}$



d. Force polygon

Scale

1 cm = 0.03 kg-m.



2 And vector do is proportional to 0.08 m_D, therefore by measurement,

$|$ 2.2×0.3 = 0.66

$0.08\, m_D$: vector do = 0.66 kg-m

$\boxed{\therefore m_D = 8.25 \text{ kg}}$

$\therefore$ From figure (c), By measurement,

7.8 × 0.03
= 0.235

$0.66x = 0.235$

$x = \frac{0.235}{0.66}$

$\boxed{x = 0.356 \text{ m}}$

3. Angular position of mass D

By measurement from figure.

$\angle BOD = 251°$ the angular position of mass at D from mass B in the anticlockwise direction.

6. A shaft has three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centres from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of metal is 7000 kg/m3. Find the amount of out-of balance force and couple at 600 r.p.m. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric. Find the amount of the masses and their angular positions.

$\underset{\sim\sim}{\text{soln:}}$

$D = 75 \text{ mm} = 0.075 \text{ m}$

$t = 25 \text{ mm} = 0.025 \text{ m}$

$r_A = 12 \text{ mm} = 0.012 \text{ m}$

$r_B = 18 \text{ mm} = 0.018 \text{ m}$

$r_C = 12 \text{ mm} = 0.012 \text{ m}$

$\rho = 7000 \text{ kg/m}^3$

$N = 600 \text{ r.p.m}$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60} = 62.84 \text{ rad/sec}.$$

$r_L = r_M = 75 \text{ mm} = 0.075 \text{ m}.$

We know that Mass of eccentric,

$m_A = m_B = m_C = \text{Volume} \times \text{Density}$

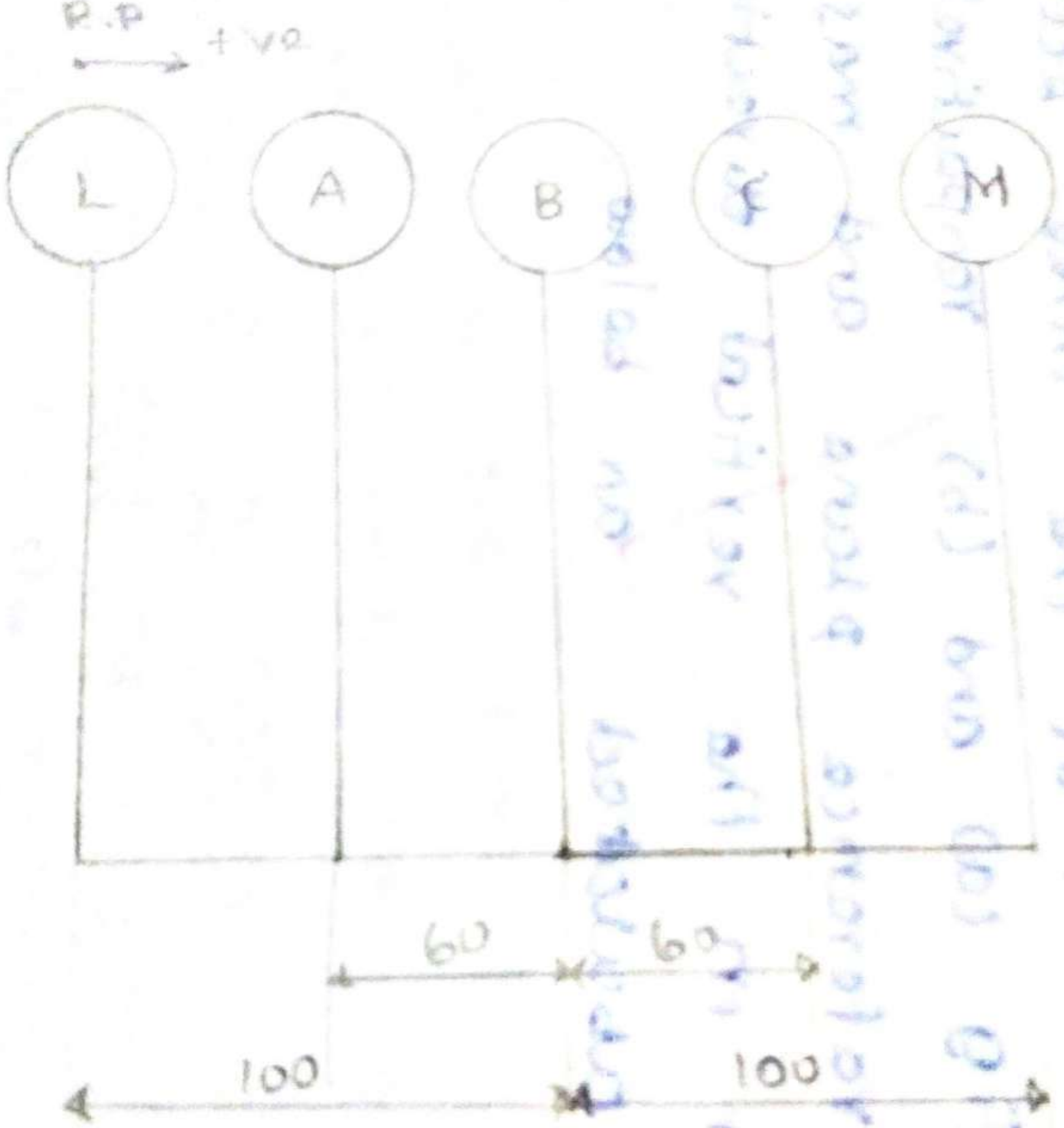
$= \pi/4 \times D^2 \times t \times \rho$

$= \pi/4 \times (0.075)^2 \times (0.025) \times 7000$

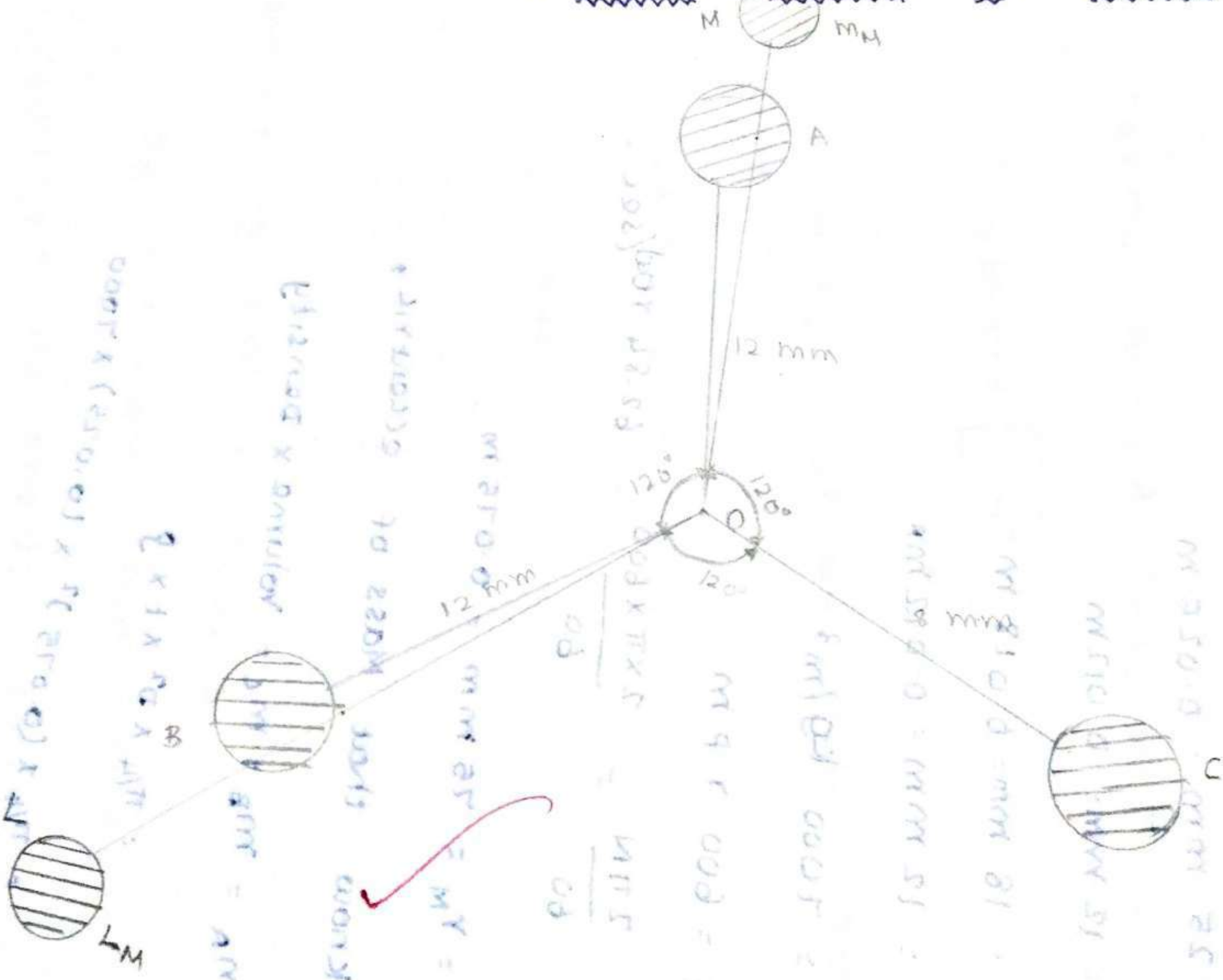
$m_A = m_B = m_C = 0.77 \text{ kg}$

Let L and M be the planes at distances of 100 mm from the central plane of middle eccentric. The position of the planes and the angular position of the three eccentrics is shown in fig (a) and (b) respectively. Assuming L as the reference plane and mass of the eccentric A in the vertical direction, the data may be tabulated as below.

a, position of planes.



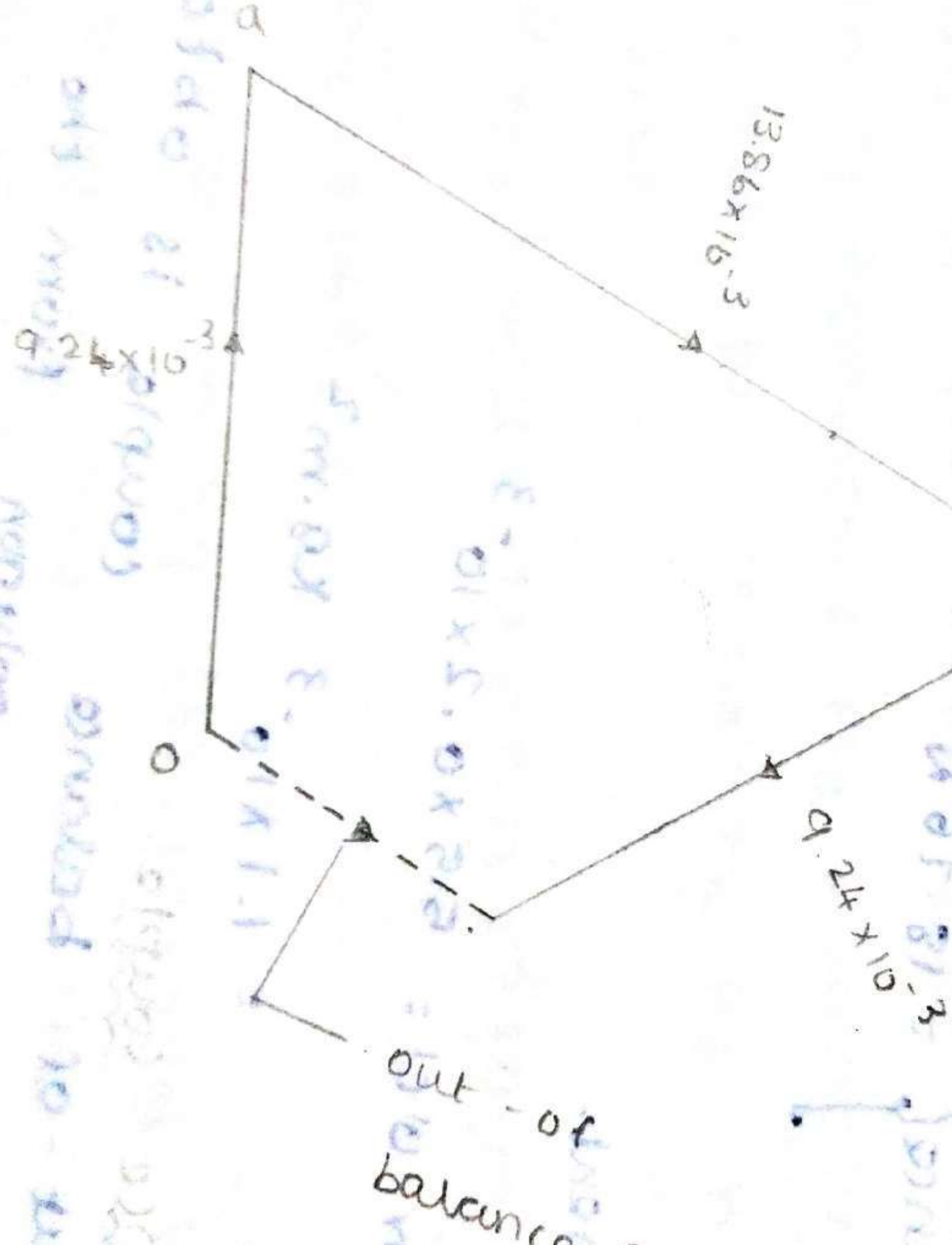
b. angular position of masses:



C. Force polygon:

scale:

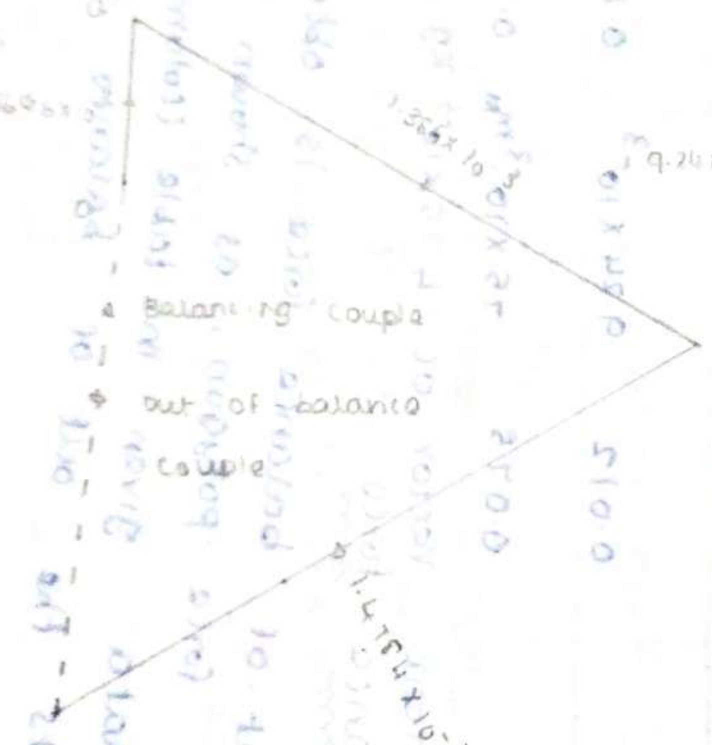
$1\,cm = 2 \times 10^{-3}$ kg m.



d. Couple polygon

Scale

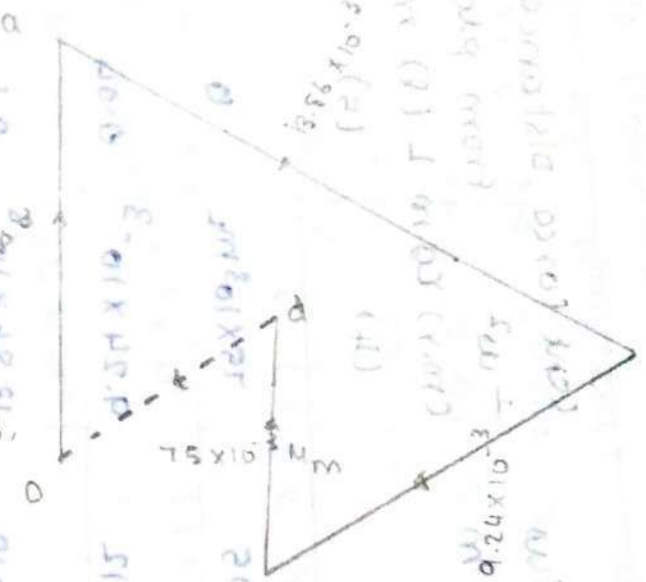
1 cm - 0.2×10^{-3} kg.m^2



e. Force polygon

Scale

1cm - 2×10^{-3} kg.m.



plane (1)	Mass (m) kg (2)	Radius (r) m (3)	cent force $\div \omega^2$ (m.r) kg.m (4)	Distance from plane L (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg.m² (6)
L (R.P)	m_L	0.075	$75\times10^{3} m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75\times10^{-3} m_M$	0.20	$15\times10^{-3} m_M$

By Measurement vector oc 4.75×10^{-3} kg m

out-of balance force

The-out-of balance force is obtained by drawing the force polygon, as shown in fig (c), from the data given in table (column 4). The resultant oc represents the out of balance force.

out of balance force $= mr\omega^2$

By measurement.

here, $mr = 4.75\times10^{-3}$ kg m

so, out of balance force $= 4.75\times10^{-3}\times\omega^2$

$= 4.75\times10^{-3}\times(62.84)^2$

out of balance force $= 18.76$ N

By Measurement-

vector o'c' : $5.5\times0.2\times10^{-3}$

$= 1.1\times10^{-3}$ kg.m²

out-of balance couple:

The out-of balance couple is obtained by drawing the couple polygon from the data given in table (column b), as shown in fig (a). The resultant o'c' represents the out-of balance couple. since the couple is proportional to the product of force and distance (m.r.l). therefore by measurement,

out-of balance couple vector $o'c' = 1.1 \times 10^{-3}$ kg.m^2

$= 1.1 \times 10^{-3} \times \omega^2 = 1.1 \times 10^{-3} \times (62.84)^2$

out of balance couple $= 4.34$ N-m

Amount of balancing masses and their angular positions:

The vector c'o' (in the direction from c' to o') as shown in fig (d) represents the balancing couple and is proportional to $15 \times 10^{-3}\, m_M$

$15 \times 10^{-3}\, m_M$ = vector $c'o' = 1.1 \times 10^{-3}$ kg.m^2

$\boxed{m_M = 0.073 \text{ kg}}$

Draw OM in fig (b) parallel to vector c'o'. By measurement, we find that the angular position of balancing mass (m_M) is 5° from mass A in the clockwise direction.

$\boxed{\theta_M = 5^\circ \text{ from mass A in clockwise direction}}$

In order to find the balancing mass (m_L), a force polygon as shown in fig. (e) is drawn. The closing side of the polygon i.e. vector do (in the direction from d to o) represents the balancing force and is proportional to $75 \times 10^{-3}\, m_L$. By measurement, we find that

$75 \times 10^{-3}\, m_L$ = vector $do = 5.2 \times 10^{-3}$ kg-m

$m_L = 0.0693$ kg

Draw an OL in fig. (b), parallel to vector do. By measurement, we find that the angular position of mass (m_L) is 124° from mass A in the clockwise direction.

7. A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pullys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg carries and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine: 1. relative angular positions of the pulleys, and 2. dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m

Given data:

$m_A = 48$ kg.

$m_C : 20$ kg

$r_A : 15$ mm $: 0.015$ m

$r_C : 12.5$ mm $: 0.0125$ m

$m_B = 56$ kg

$r_B = 15$ mm $= 0.015$ m

$N - 300$ r.p.m

$\omega = \dfrac{2 \times \pi \times 300}{60} = 31.42$ rad/sec.

Soln:

1. The position of shaft and pulley's shown in Figure (A).

Let, m_L and m_N = Mass at the bearings L & N

r_L and r_M = radius of rotation of the masses at L & N respectively.

Assuming the plane of bearing L as reference plane.

It is assumed that the mass of pully B acts in vertical direction

plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force $\div \omega^2$ (m.r) kg m (4)	Distance from Ref plane L (l) m (5)	Couple $\div \omega^2$ (m.r.l) kg.m^2 (6)
A	48	0.015	0.72	-0.45	-0.324
L	m_L	r_L	$m_L r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M r_m$	1.8	1.8 $m_M r_m$
C	20	0.0125	0.25	2.25	0.5625

Now in figure (b) draw OA parallel to vector bc and oc parallel to vector co. By measurement, we find that

Angle between pulleys B and A = 160°

Angle between pulleys A and C = 75°

Angle between pulleys C and B = 125°.

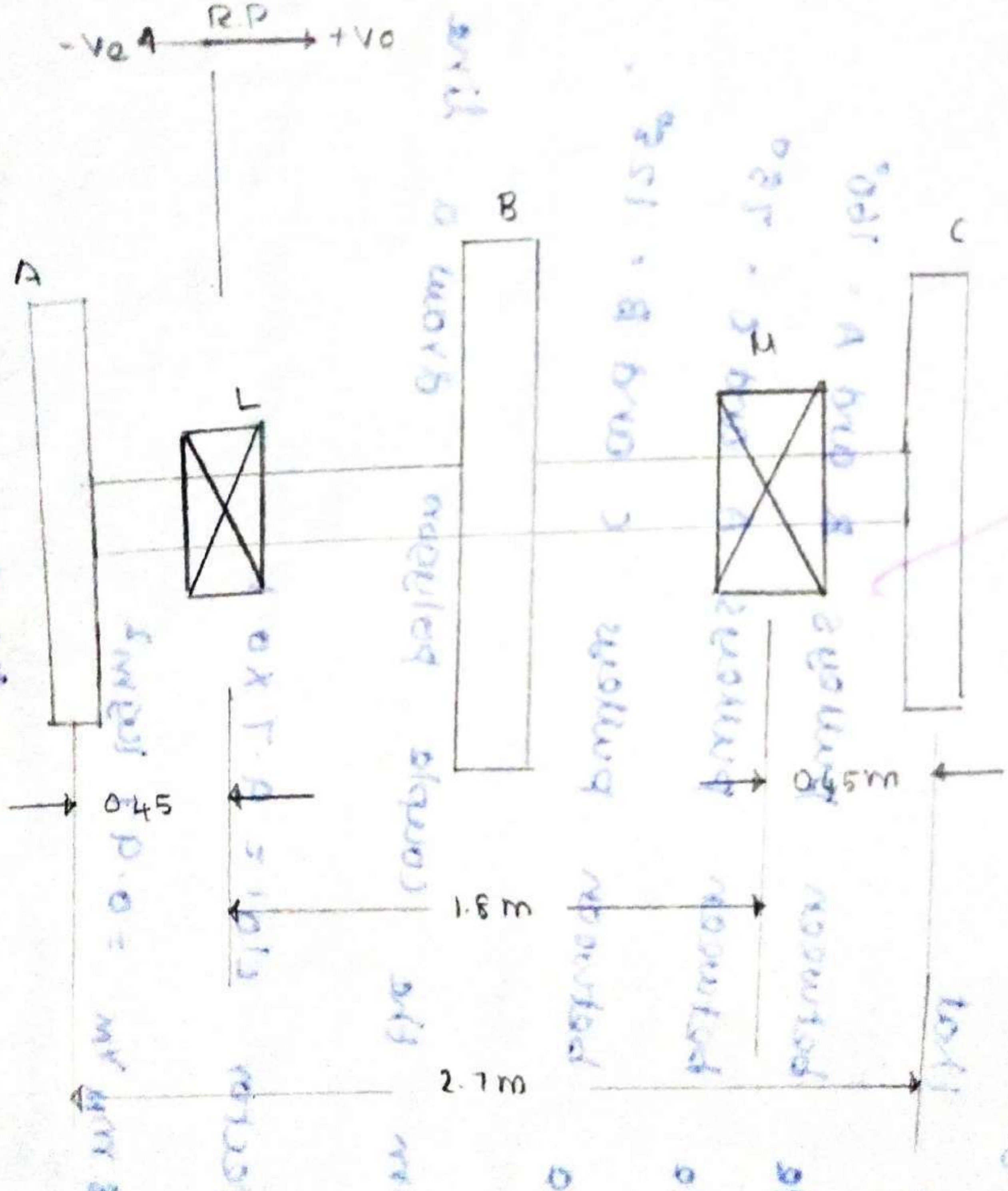
From the couple polygon draw a line c'o'

vector $c'o' = 9.7 \times 0.1$

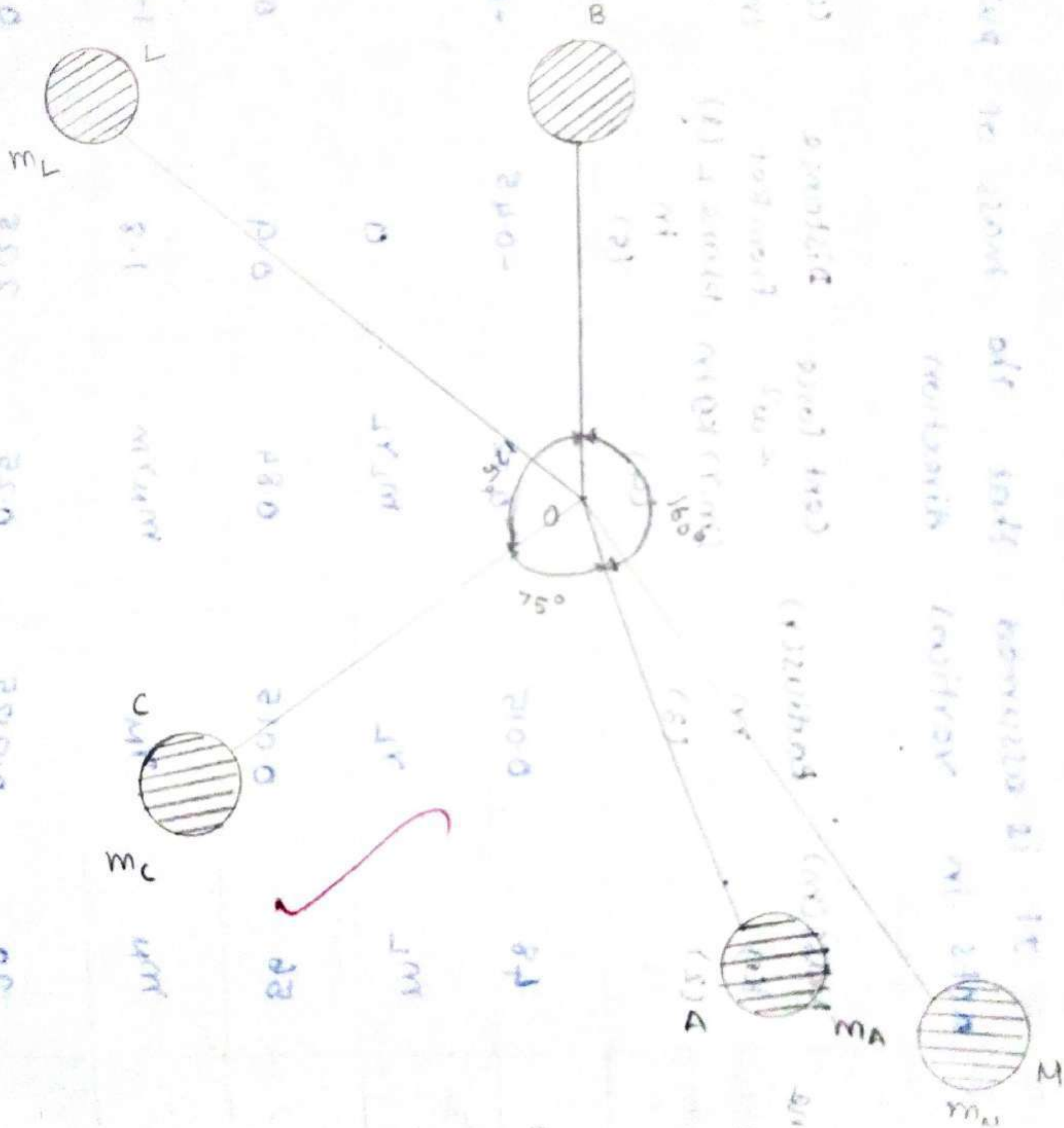
$1.8\, m_M r_m = 0.97\ kgm^2$

$m_M r_m = 0.54$ kg m.

a, position of shaft and pulleys:



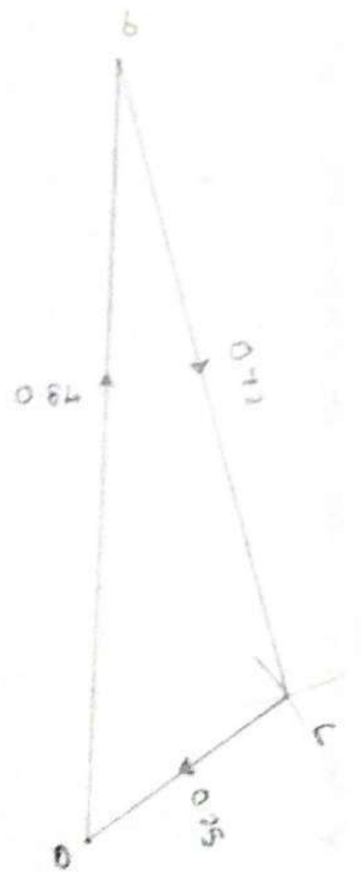
b. Angular position of pulleys :



Force polygon

Scale

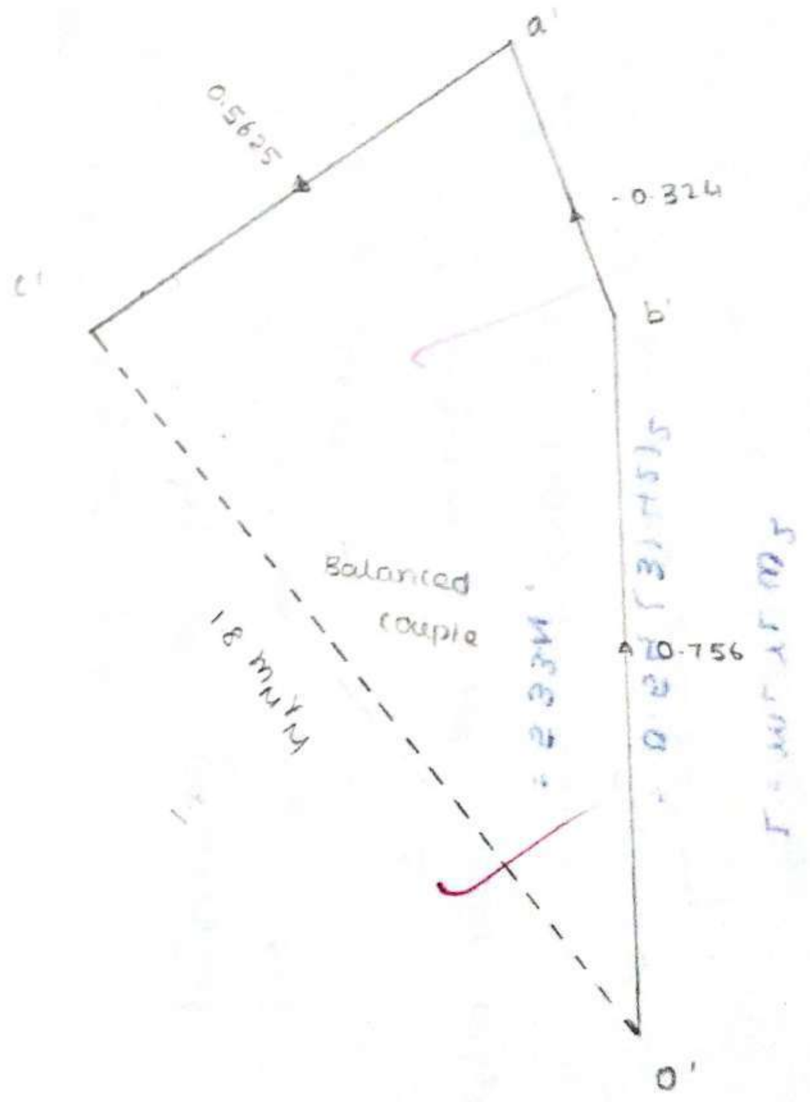
1cm = 0.1 kg·m.



d. couple polygon

scale

1 cm = 0.1 kg·m²



a. Force polygon

scale

$1\ cm = 0.1\ kg.m^2$

1. Dynamic forces produced on the bearings:

Dynamic forces on the bearing M:

$M = m_M r_M \omega^2$

$= 0.54 (31.42)^2$

$= 533\ N$

Dynamic forces on the bearing L,

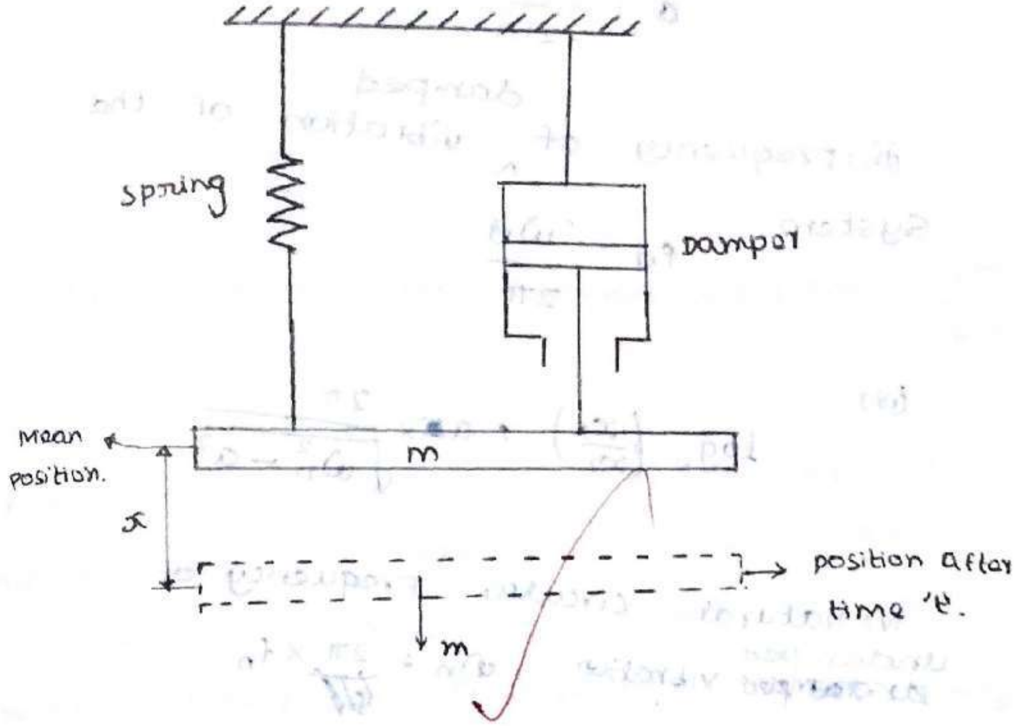
$L = m_L r_L \omega^2$

$= 0.54 (31.42)^2$

$= 533 N.$

Frequency of Free Damped vibration (viscous Damping)

The motion of a body is resistant by Frictional Forces. In vibrating systems the effect of Friction is refer to as Damping. The Damping provided by fluid resistance is known as viscous Damping.



Damping factor (or) Damping ~~racto~~ ratio :

The ratio of actual Damping coefficient (c) to the critical Damping coefficient (c_c) is known as Damping factor (or) Damping ratio.

Mathematically,

Damping factor $= \frac{c}{c_c}$

Where,

$c_c = 2m\omega_n$.

Formulae

(i) circular Frequency of undamped vibrations $\omega_n = \sqrt{\frac{s}{m}}$

(ii) circular Frequency of damped vibrations $\omega_d = \sqrt{\omega_n^2 - a^2}$

Where,

$$a = \frac{c}{2m}$$

(iii) Frequency of damped vibration of the system $f_d = \frac{\omega_d}{2\pi}$

(iv)

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{\omega_n^2 - a^2}}$$

(v) Natural circular Frequency of undamped ~~undamped~~ vibration $\omega_n = 2\pi \times f_n$

(vi) Natural circular Frequency of damped vibration $\omega_d = 2\pi \times f_d$

(vii) logarithmic decrement,

$$\delta = \frac{2\pi c}{\sqrt{c_c^2 - c^2}}$$

(viii) Critical damping co-efficient $C_c = 2m\omega_n$.

(ix) Damping factor $= \frac{c}{c_c}$

(x) $\delta = \log_e \left[\frac{x_n}{x_{n+1}} \right]$.

(xi) $\log_e \left[\frac{x_1}{x_2} \right] = a \times t_p$.

(xii) $f_d = \frac{1}{2\pi} \sqrt{\omega_n^2 - a^2} = \frac{1}{t_p}$

(xiii) $f_n = \frac{\omega_n}{2\pi}$

(xiv) periodic (or) damped vibration. $= \frac{2\pi}{\omega_d}$

1. A vibrating system consist of a mass of 200 kg, a spring of stiffness 80 N/mm and a damper with damping co-efficient of 800 N/m/sec determine the frequency of vibration of the system.

given data:

$m = 200$ kg

$s = 80$ N/mm

$s = 80 \times 10^3$ N/m

$c = 800$ N/m/sec

W.K.T

(i) Circular frequency of undamped vibration $\omega_n = \sqrt{\frac{s}{m}}$

$$= \sqrt{\frac{80 \times 10^3}{200}}$$

$$\omega_n = 20 \text{ rad/sec.}$$

(ii) Circular Frequency of damped vibration $\omega_d = \sqrt{\omega_n^2 - a^2}$

$$= \sqrt{20^2 - a^2}$$

here,

$$a = \frac{c}{2m}$$

$$a = \frac{800}{2 \times 200}$$

$$a = 2$$

$$\omega_d = \sqrt{20^2 - 2^2}$$

$$\boxed{\omega_d = 19.9 \text{ rad/sec}}$$

(iii) Frequency of damped vibration of the system.

$$f_d = \frac{\omega_d}{2\pi}$$

$$= \frac{19.9}{2 \times \pi}$$

$$\boxed{f_d = 3.16 \text{ Hz}}$$

27/7/16

2. The Following data are given for a vibratory system with viscous damping.

Mass = 2.5 kg. spring constant = 3 N/mm and the amplitude decreases to 0.25 of the initial value after 5 consecutive cycles. determine damping co-efficient of the damper in the system.

given data

$m = 2.5\ kg$

$s = 3\ N/mm$

$s = 3 \times 10^3\ N/m$

$x_6 = 0.25\ x_1$

W.K.T.

Natural Frequency of vibration, $\omega_n = \sqrt{\frac{s}{m}}$

$$= \sqrt{\frac{3 \times 10^3}{2.5}}$$

$\omega_n = 34.64$ rad/sec.

W.K.T.

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}$$

$$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6}$$

$$\frac{x_1}{x_6} = \left(\frac{x_1}{x_2}\right)^5$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5}$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{0.25x_1}\right)^{1/5}$$

$$= (4)^{1/5}$$

$$\frac{x_1}{x_2} = 1.32$$

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e(1.32) = a \times \frac{2\pi}{\sqrt{(34.64)^2 - a^2}}$$

$$\frac{2\pi}{\sqrt{1198.544 - a^2}} \times a = \log_e(1.32)$$

$$0.1206 = \frac{2\pi \times a}{\sqrt{1198.544 - a^2}}$$

Take square on both sides,

$$\frac{4\pi^2 \times a^2}{1198.544 - a^2} = 0.0145$$

$$1198.544 - a^2 = 2722.64\, a^2$$

$$2722.64a^2 + a^2 = 1198.544$$

$2 \times 123.64\, a^2 = 1198.544$

$a^2 \times$

$$0.1206 \times \sqrt{1198.544 - a^2} = 2\pi \times a$$

Take square on both sides

$$0.01451 \times 1198.544 - a^2 = 6.28 \times a \quad 39.48 a^2$$

$$39.48 a^2 + a^2 = 17.38$$

$$40.48 a^2 = 17.38$$

Take square on both sides.

$$0.0442^2 \times (34.64^2 - a^2) = a^2$$

$$1.952 \times 10^{-3} \times (34.64^2) - 1.952 \times 10^{-3} a^2 = a^2$$

$$1.952 \times 10^{-3} (34.64)^2 = a^2 + 1.952 \times 10^{-3} a^2$$

$$2.343 = a^2 (1 + 1.952 \times 10^{-3})$$

$$1.002\, a^2 = 2.343$$

$$a^2 = \frac{2.343}{1.002}$$

$$a^2 = 2.338$$

$$\boxed{a = 1.529}$$

$$a = c/2m$$

$$c = a \times 2m$$

$$c = 1.529 \times (2 \times 2.5)$$

$$\boxed{c = 7.646 \text{ N/m/s}}$$

1. An instrument vibrates with a frequency of 1 Hz when there is no damped. When damping is provided frequency of damped vibration to be 0.9 Hz

Find

(i) damping Factor

(ii) Logarithmic decrement.

Given data:

$f_n = 1\ Hz$

$f_d = 0.9\ Hz$.

Soln:

Damping factor

Let

$m \rightarrow$ Mass of the instrument (Kg)

$c \rightarrow$ damping co-efficient

$c_c \rightarrow$ critical damping coefficient

$\frac{c}{c_c} \rightarrow$ damping factor

$\omega_n = 2\pi \times f_n$

$\boxed{\omega_n = 6.284\ rad/sec}$ $\left[\because f_n = \frac{1}{2\pi}(\omega_n)\right]$

$\omega_d = 2\pi \times f_d$

$\boxed{\omega_d = 5.66\ rad/sec}$

we also know that circular frequency of damped vibration, (ω_d)

$$\omega_d = \sqrt{\omega_n^2 - a^2}$$

$$5.66 = \sqrt{6.284^2 - a^2}$$

Take squares on both sides

$$6.284^2 - a^2 = 32.0356$$

$$a^2 = 39.48 - 32.0356$$

$$a^2 = 7.45$$

$$a = 2.73$$

W.K.T.

$$a = c/2m$$

$$c = a(2m)$$

$$c = 2.73 \times 2m$$

$$\boxed{c = 5.46m\ N/m/s}$$

$$C_c = 2m\omega_n$$

$$= 2m \times 6.284$$

$$\boxed{C_c = 12.568\ m\ N/m/s}$$

∴ Damping factor $= \frac{c}{C_c}$

$$= \frac{5.48\ m}{12.568\ m}$$

$$\boxed{\text{Damping factor} = 0.436}$$

(ii) Logarithmic decrement :

$$\delta = \frac{2\pi c}{\sqrt{c_c^2 - c^2}}$$

$$\delta = \frac{2 \times \pi \times 5.46 m}{\sqrt{(12.568 m)^2 - (5.46 m)^2}}$$

$\delta =$

$$\sqrt{(12.568 m)^2 - (5.46 m)^2}\,\delta = 2 \times \pi \times 5.46 m$$

Take square on both sides.

$$(157.95 m^2 - 29.812 m^2) \times \delta^2 = 1176.92 m^2$$

$$128.138 m^2 \times \delta^2 = 1176.92 m^2$$

$$\delta^2 = 9.18$$

$$\boxed{\delta = 3.03}$$

1. The Measurements on a mechanical vibrating system so at it has mass of 8 kg and that the springs can be combined to give an equivalent spring of stiffness 5.4 N/mm If the vibrating system have a dashpot (piston cylinder arrangement) attached which excerts a force of 40N, when the mass has a velocity of 1 m/sec Find

(i) critical damping co-efficient

(ii) damping factor

(iii) logarithmic decrement

(iv) Ratio of two consecuting amplitudes

Given data:

$m = 8\ kg$

$S = 5.4\ N/mm$

$S = 5.4 \times 10^3\ N/m$

Since the force is excerted by dash pot is 40 N and the mass has a velocity of 1 m/sec. Therefore damping co-efficient $\boxed{c = \frac{40}{1}\ N/m/s}$

(i) critical damping co-efficient (C_c)

$$C_c = 2 m \omega_n$$

$$= 2 \times 8 \times \sqrt{\frac{S}{m}}$$

$$= 2 \times 8 \times \sqrt{\frac{54 \times 10^3}{8}}$$

$$\boxed{C_c = 415.692\ N/m/s}$$

(ii) damping factor $= \frac{c}{C_c}$

$$= \frac{40}{415.692} = 0.096.$$

Damping factor $= 0.096$

(iii) Logarithmic decrement

$$\delta = \frac{2\pi c}{\sqrt{c_c^2 - c^2}}$$

$$\delta = \frac{2 \times \pi \times 40}{\sqrt{415.692^2 - 40^2}}$$

$$\boxed{\delta = 0.6}$$

(iv) Ratio of two consecutive amplitude:

Let

x_n & $x_{n+1} \rightarrow$ magnitude of two consecutive amplitude.

$$\delta = \log_e \left[\frac{x_n}{x_{n+1}} \right]$$

$$e^{\delta} = \frac{x_n}{x_{n+1}}$$

$$e^{0.6} = \frac{x_n}{x_{n+1}}$$

$$\boxed{\frac{x_n}{x_{n+1}} = 1.82}$$

1. A mass suspended from helical spring vibrate in a viscous fluid medium. whose resistance varies directly with the speed which absorb that the frequency of damped vibration is 90/min and the amplitude decreases to 20% of its initial value in one complete revolution vibration. Find the frequency of the free un-damped vibration of the system.

Given data:

$$f_d = 90/min$$

$$= 90/60$$

$$f_d = 1.5 \ Hz$$

Time period, $t_p = \frac{1}{f_d}$

$$= \frac{1}{1.5}$$

$$\boxed{t_p = 0.67 s}$$

Let,

$x_1 \rightarrow$ Initial amplitude

$x_2 \rightarrow$ Final amplitude

After one complete vibration.

$$x_2 = 20\% . x_1$$

$$x_2 = 0.2 x_1$$

Now,

$$\log_e \left(\frac{x_1}{x_2}\right) = a t_p .$$

$$a = \frac{1}{t_p} \wedge \log_e \left(\frac{x_1}{0.2 x_1} \right)$$

$$a = \frac{1}{0.67} \times \cancel{0.699}\, 1.609$$

$$\boxed{a = 2.402}$$

Frequency of damped vibration.

$$f_d = \frac{1}{2\pi} \sqrt{\omega_n^2 - a^2}$$

$$\sqrt{\omega_n^2 - a^2} = 2\pi \times f_d$$

Take square on both sides.

$$\omega_n^2 - a^2 = 4\pi^2 \times f_d^2$$

$$\omega_n^2 - 5.77 = 39.47 \times 2.25$$

$$\omega_n^2 = 94.5775$$

$$\boxed{\omega_n = 9.726 \text{ rad/sec}}$$

Frequency of un damped vibration,

$$f_n = \frac{\omega_n}{2\pi}$$

$$= \frac{9.726}{2\pi}$$

$$\boxed{f_n = 1.55 \text{ Hz}}$$

1. A mass of single degree damped vibration system 7.5 kg and makes 24 free oscillations in 14 sec. The amplitude of vibration reduces to 0.25 of its initial value after 5 oscillations. determine

(i) stiffness of the spring

(ii) logarithmic decrement

(iii) damping factor.

Given data:

$x_6 = 0.25 x_1$

$m = 7.5$ kg

Since 24 oscillations are made in 14 sec. Therefore frequency of free vibration.

$f_n = 24/14 = 1.7$ Hz.

$\omega_n = 2\pi \times f_n$

$\omega_n = 10.7$ rad/sec

1. stiffness of the spring:

Let.

s - stiffness of the spring in N/m

$$\omega_n = \sqrt{\frac{s}{m}}$$

$$10.7 = \sqrt{\frac{s}{7.5}}$$

Square on both sides.

$$\frac{S}{7.5} = 114.49$$

$$\boxed{S = 858.675 \text{ N/m}}$$

2. Logarithmic decrement:

Let,

$x_1 \Rightarrow$ Initial amplitude

$x_6 \Rightarrow$ Final amplitude after 5 oscillation.

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}$$

$$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6}$$

So $$\frac{x_1}{x_6} = \left(\frac{x_1}{x_2}\right)^5$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5}.$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{0.25\, x_1}\right)^{1/5}.$$

$$= (4)^{1/5}$$

$$\frac{x_1}{x_2} = 1.32.$$

Logarithmic decrement,

$$\delta = \log_e \left(\frac{x_1}{x_2} \right)$$

$$= \log_e (1.32)$$

$$\delta = 0.28 .$$

3. Damping factor:

Logarithmic decrement,

$$\delta = \frac{a \times 2\pi}{\sqrt{\omega_n^2 - a^2}}$$

$$0.28 = \frac{a \times 2\pi}{\sqrt{10.7^2 - a^2}}$$

$$\sqrt{10.7^2 - a^2} = \frac{a \times 2\pi}{0.28}$$

Take square on both sides.

$$10.7^2 - a^2 = \cancel{140.49}\ 503.55\, a^2$$

$$504.55\, a^2 = 114.49$$

$$a^2 = 0.227$$

$$\boxed{a = 0.476}$$

w.k.T,

$$a = \frac{c}{2m}$$

$$\therefore c = a \times 2m$$

$$c = 0.476 \times 2 \times 7.5$$

$$c = 7.2 \text{ N/m/s}$$

w.k.T,

$$c_c = (2m)\,\omega_n$$

$$c_c = 160.5 \text{ N/m/s}$$

$$\text{Damping factor} = \frac{c}{c_c}$$

$$= \frac{7.2}{160.5}$$

$$\left.\begin{array}{l}\text{Damping}\\ \text{factor}\end{array}\right\} = 0.045$$

Result.

(i) Stiffness of the spring is = 858.675 N/m

(ii) Logarithmic decrement, $\delta = 0.28$

(iii) Damping factor = 0.045.

Torsional vibrations

1. The disc move in a circle about the axis of a shaft then the vibrations are known as torsional vibrations. In this case the shaft is twisted and untwisted alternately and torsional shear stresses are induced in the shaft.

Natural Frequency of free torsional vibrations:

EM consider a shaft of negligible mass whose one end is fixed and the other end carrying a disc as shown in figure

Let,

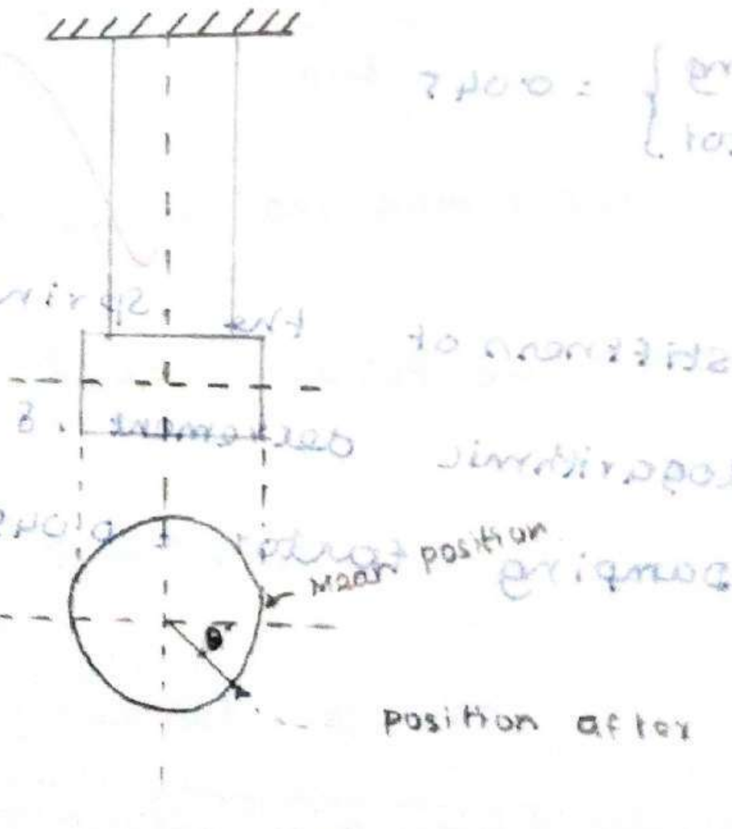
$\theta \rightarrow$ Angular displacement of the shaft from mean position after time t in radians.

$m \rightarrow$ mass of the disc in kg.

$I \rightarrow$ mass moment of Inertia of disc in $kg.m^2 = mk^2$

$k \rightarrow$ radius of gyration in metres.

$q \rightarrow$ Torsional stiffness of the shaft in Nm.



Restoring Force = $q\theta$ $\rightarrow$ (i)

and

accelerating force = $I \times \frac{d^2\theta}{dt^2}$ $\rightarrow$ (ii)

Equating equations (i) and (ii), the equation of motion is

$$I \times \frac{d^2\theta}{dt^2} = -q.\theta$$

$$I \times \frac{d^2\theta}{dt^2} + q.\theta = 0.$$

$$\therefore \frac{d^2\theta}{dt^2} + \frac{q}{I} \times \theta = 0 \rightarrow \text{(iii)}$$

The fundamental equation of the simple harmonic motion is

$$\frac{d^2\theta}{dt^2} + \omega^2 x = 0 \rightarrow \text{(iv)}$$

comparing equations (iii) and (iv)

$$\omega = \sqrt{\frac{q}{I}}$$

$\therefore$ Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

and natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

The value of the torsional stiffness q may be obtained from the torsion equation.

$$\frac{T}{J} = \frac{C\theta}{l} \quad (or) \quad \frac{T}{\theta} = \frac{C.J}{l}$$

$$q = \frac{C.J}{l}$$

where

C = modulus of rigidity for the shaft material

J = polar moment of inertia of the shaft cross section

$= \pi/32 \times d^4$

d - diameter of the shaft.

l - length of the shaft.

1. A shaft of 100 mm diameter and 1 metre long has one of its end fixed and the other end carries a disc of mass 500 kg at a radius of gyration of 450 mm. The modulus of rigidity for the shaft material is 80 GN/m². Determine the frequency of torsional vibrations.

Soln:

Given data

d = 100 mm = 0.1 m

l = 1 m ;

m = 500 kg

k = 450 mm = 0.45 m

C = 80 GN/m² = 80×10^9 N/m²

We know that polar moment of inertia of the shaft,

$$J = \pi/32 \times d^4 = \pi/32 \times (0.1)^4 = 9.82 \times 10^{-6} \, m^4.$$

∴ Torsional stiffness of the shaft,

$$q = \frac{CJ}{l} = \frac{80 \times 10^9 \times 9.82 \times 10^{-6}}{1}$$

$$\boxed{q = 785.6 \times 10^3 \text{ N-m}}$$

We know that mass moment of inertia of the shaft.

$$J = m.k^2 = 500(0.45)^2$$

$$= 101.25 \ kg.m^2$$

∴ Frequency of torsional vibrations

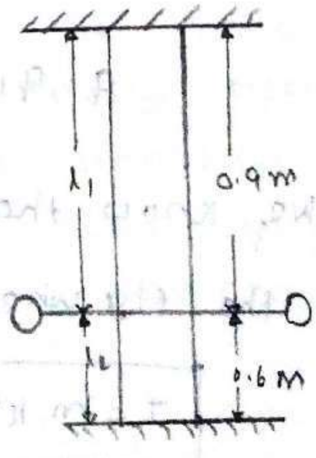
$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{J}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{785.6\times10^3}{101.25}} = \frac{88.1}{2\pi}$$

$$\boxed{f_n = 14 \ Hz}$$

2, A flywheel is mounted on a vertical shaft as shown in fig. The both ends of a shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg and its radius of gyration is 0.5 m. Find the natural Frequency of torsional vibrations. if the modulus of rigidity for the shaft material is 80 GN/m².

Given data:



d = 50 mm = 0.05 m

m = 500 kg

k = 0.5 m

G = 80 GN/m²

$G = 8\cancel{0}4\times10^9 \ N/m^2$

Soln:

We know that polar moment of Inertia of the shaft,

$$J = \pi/32 \times d^4 = \pi/32 \times (0.05)^4 \, m^4$$

$$J = 0.6 \times 10^{-6} m^4$$

∴ Torsional stiffness of the shaft for length l_1,

$$q_1 = \frac{C.J}{l_1} = \frac{84 \times 10^9 \times 0.6 \times 10^{-6}}{0.9}$$

$$q_1 = 56 \times 10^3 \; N\text{-}m$$

∴ Torsional stiffness of the shaft for length l_2.

$$q_2 = \frac{C.J}{l_2} = \frac{84 \times 10^9 \times 0.6 \times 10^{-6}}{0.6}$$

$$\boxed{q_2 = 84000 \; N\text{-}m}$$

∴ Total torsional stiffness of the shaft

$$q = q_1 + q_2 = 56 \times 10^3 + 84000 = 140 \times 10^3 \; N\text{-}m.$$

We know that mass moment of Inertia of the fly wheel,

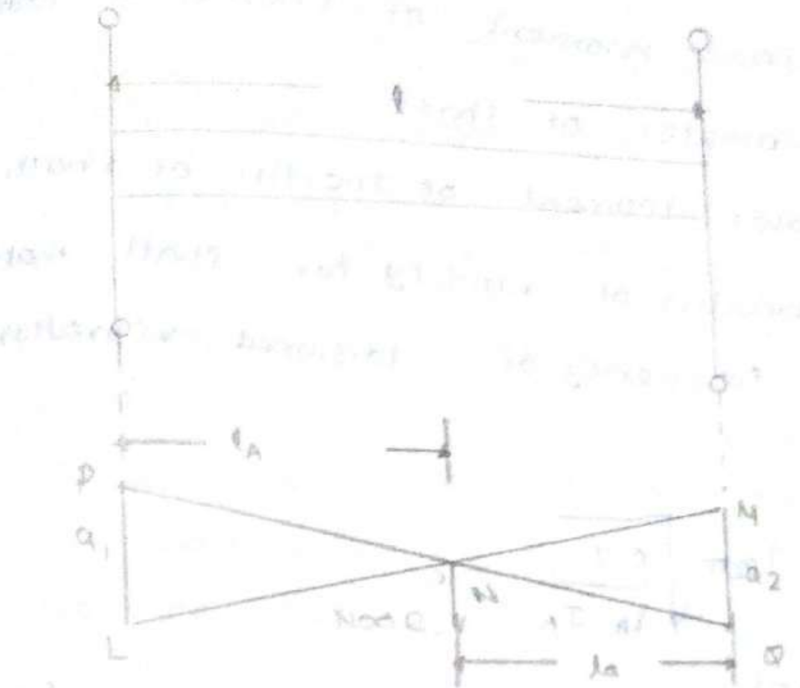
$$\boxed{I = m.k^2 = 500(0.5)^2 = 125 \; kg.m^2}$$

∴ Natural frequency of torsional vibration,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}} = \frac{1}{2\pi}\sqrt{\frac{140 \times 10^3}{125}}$$

$$= \frac{33.5}{2\pi} = 5.32 \; Hz.$$

Free torsional vibrations of a two Rotor system



*Consider a two rotor system as shown in fig. It consists of a shaft with two rotors at its ends. In this system, the torsional vibrations occur only when the two rotors A and B move in opposite directions i.e If A moves in anticlockwise direction then B moves in clockwise direction at the same instant and vice versa. It may be noted that the two rotors must have the same frequency.

We see from fig that the node lies at point N. This point can be safely assumed as a fixed end and the shaft may be considered as two separate shafts NP and NQ each fixed to one of its ends and carrying rotors at the free ends.

Let.

l - length of shaft.

l_A - length of part NP i.e distance of node from rotor A.

l_B = length of part NQ i.e distance of node from rotor B.

I_A = Mass moment of Inertia of rotor A

I_B = Mass moment of Inertia of rotor B.

d = Diameter of shaft.

J = polar moment of Inertia of shaft, and

C = Modulus of rigidity for shaft material

Natural frequency of torsional vibration for rotor A.

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_A.I_A}}$$

and Natural frequency of torsional vibration for rotor B

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_B.I_B}}$$

Since,

$f_{nA} = f_{nB}$; therefore

$$\frac{1}{2\pi}\sqrt{\frac{CJ}{l_A.I_A}} = \frac{1}{2\pi}\sqrt{\frac{C.J}{l_B.I_B}}$$

$$\frac{1}{\sqrt{l_A.I_A}} = \frac{1}{\sqrt{l_B.I_B}}$$

Take square on both sides

$$\frac{1}{l_A.I_A} = \frac{1}{l_B.I_B}$$

$\therefore l_B . I_B = l_A . I_A \longrightarrow (3)$

$$\therefore l_A = \frac{l_B . I_B}{I_A}$$

we also know that,

$l = l_A + l_B \rightarrow (4)$.

From equations (iii) and (iv), we may find the value of l_A and l_B and hence the position of node substituting the values of l_A or l_B in equation (i) or (ii), the natural frequency of torsional vibration for a two rotor system may be evaluated.

(2M)

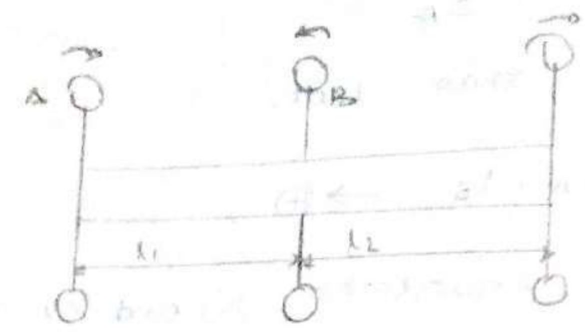
elastic line

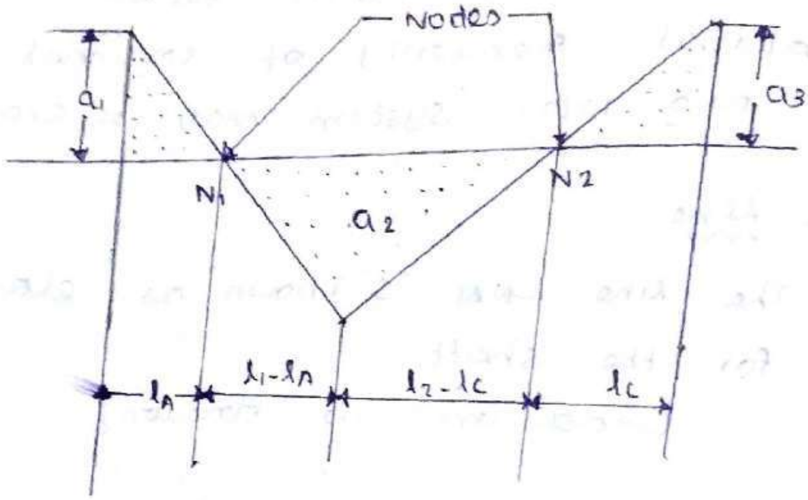
The line LNN is known as elastic line for the shaft.

[Diagram is drawn]

Free torsional vibrations of a three Rotor system:

consider a three rotor system as shown in fig. It consists of a shaft and three rotors A, B and C. The rotors A and C are attached to the ends of a shaft. whereas the rotor B is attached in between A and C. The torsional vibrations may occur in two ways, that is with either one node or two nodes. In each case, the two rotors rotate in one direction and the third rotor rotates in opposite direction with the same frequency. Let the rotors A and C of the system shown in fig rotate in the same direction and the rotor B in opposite direction.





Let

l_1 = Distance between rotors A and B,

l_2 = Distance between rotors B and C.

l_A = Distance of Node N_1 from rotor A.

l_C = Distance of Node N_2 from rotor C.

I_A = Mass moment of Inertia of rotor A

I_B = Mass moment of Inertia of rotor B

I_C = Mass moment of Inertia of rotor C.

d = Diameter of shaft.

J = polar moment of Inertia of shaft, and

C = modulus of rigidity for shaft material

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A \cdot I_A}}$$

Natural frequency of torsional vibrations for rotor B

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_B}\left(\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_C}\right)} \rightarrow (2).$$

Natural frequency of torsional vibrations for rotor C.

$$f_{nC} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_C I_C}} \rightarrow (3).$$

equating (i) and (iii) we get.

$$\frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_C I_C}}.$$

$$l_A I_A = l_C I_C$$

$$l_A = \frac{l_C I_C}{I_A} \rightarrow (4).$$

equating (2) & (3),

$$\frac{1}{2\pi}\sqrt{\frac{CJ}{I_B}\left(\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_C}\right)} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_C I_C}}$$

(or)

$$\frac{1}{I_B}\left(\frac{1}{l_1 - l_A} + \frac{1}{l_2 - l_C}\right) = \frac{1}{l_C I_C} \rightarrow (5).$$

Problem.

A steel shaft ABCD 1.5 m long has flywheel at its ends A and D. The mass of flywheel D is 800 kg and has a radius of gyration of 0.9 m. The connecting shaft has a diameter of 50 mm for the portion AB which is 0.4 m long; and has a diameter of 60 mm for the portion BC which is 0.5 m long; and has diameter 'd' mm for the portion on which is 0.6 m long Determine

1, The diameter 'd' of the portion CD so that the nodes of the torsional vibration of the system will be at the centre of the length BC. and

the natural frequency of the torsional vibrations.

The modulus of rigidity for the shaft material is 80 GN/m².

given data:

$L = 1.5\ m$

$m_A : 600\ kg$

$k_A : 0.6\ m$

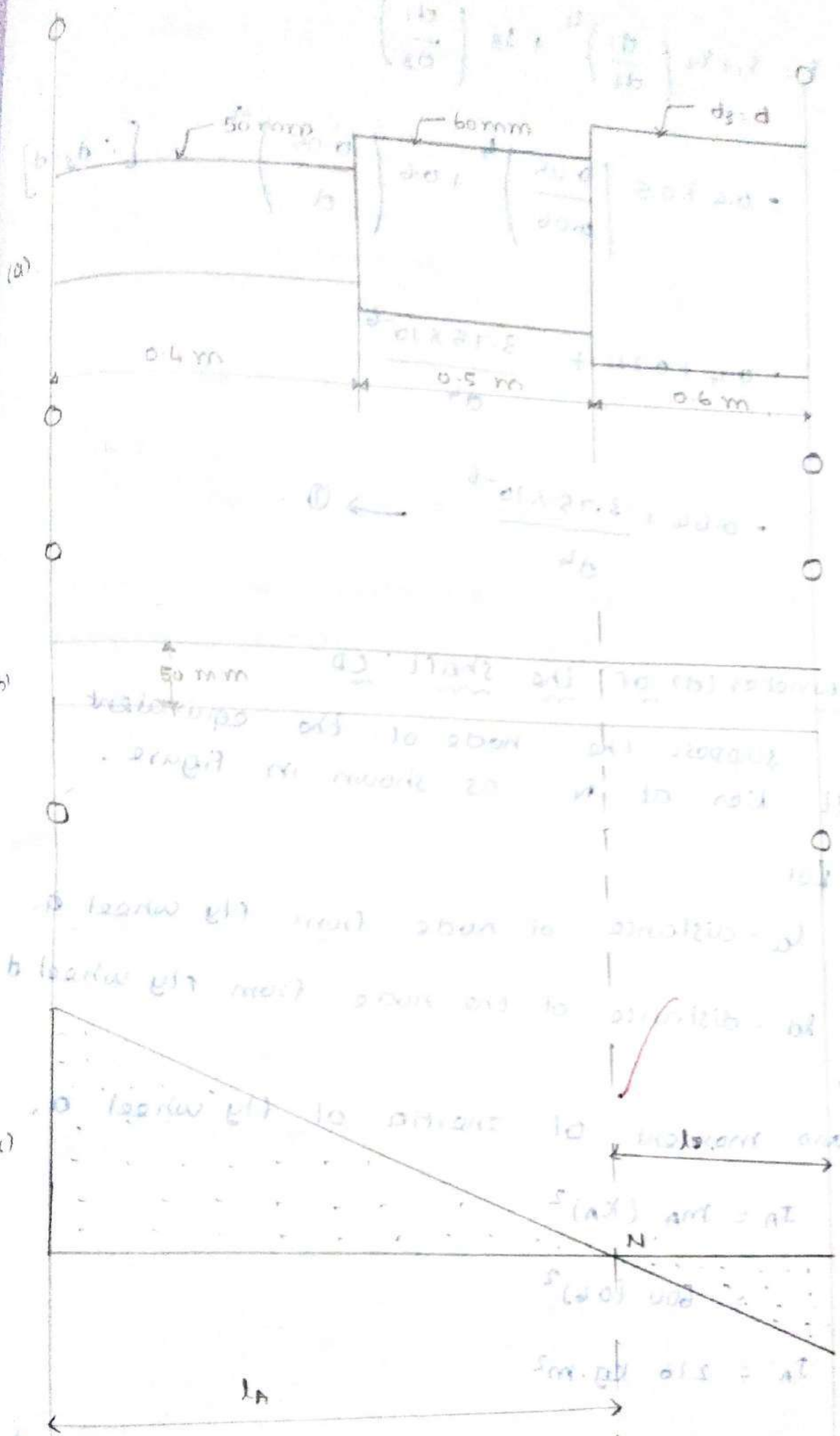
$m_D : 800\ kg$

$d_1 = 50\ mm : 0.05\ m$ $\quad l_1 = 0.4\ m$

$d_2 : 60\ mm : 0.06\ m$ $\quad l_2 = 0.5\ m$

$d_3 = d$ $\quad l_3 = 0.6\ m$

$C = 80\ GN/m^2 = 80 \times 10^9\ N/m^2$.


(a)
50 mm
60 mm
$d_3 = d$
0.4 m
0.5 m
0.6 m
(b)
50 mm
(c)
N
l_B
l_A

$$l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

$$= 0.4 + 0.5\left(\frac{0.05}{0.06}\right)^4 + 0.6\left(\frac{0.05}{d}\right)^4 \qquad [\because d_3 = d]$$

$$= 0.4 + 0.24 + \frac{3.75 \times 10^{-6}}{d^4}$$

$$= 0.64 + \frac{3.75 \times 10^{-6}}{d^4} \longrightarrow (1).$$

(i) diameter (d) of the shaft : CD

suppose the node of the equivalent shaft lies at N as shown in figure.

Let,

l_a - distance of node from fly wheel a.

l_d - distance of the node from fly wheel d.

W.K.T.

Mass moment of Inertia of fly wheel a.

$$I_A = m_A (K_A)^2$$

$$= 600 (0.6)^2$$

$$I_A = 216 \text{ kg.m}^2$$

Mass moment of Inertia of fly wheel d.

$$I_D = m_D (K_D)^2$$

$$= 800 \times 0.9^2$$

$$\boxed{I_D = 648 \text{ kg m}^2}$$

W.K.T.

$$I_A l_A = I_D l_D$$

$$l_D = \frac{l_A I_A}{I_D}$$

$$= \frac{l_A \times 216}{648}$$

$$l_D = \frac{l_A}{3}$$

$$l_D = 0.333\, l_A \rightarrow ②$$

Since, the node lies the centre of the length BC in an original system.

Its equivalent length from rotor A.

$$l_A = l_1 + \frac{l_2}{2}\left(\frac{d_1}{d_2}\right)^4$$

$$= 0.4 + \frac{0.5}{2}\left(\frac{0.05}{0.06}\right)^4$$

$$\boxed{l_A = 0.52\ m}$$

Sub in ②

$$\therefore l_D = 0.333 \times 0.52$$

$$\boxed{l_D = 0.173\ m}$$

W.K.T,

$$l = l_A + l_D$$

Subs eqn ① for 'l'.

$$0.64 + \frac{3.75 \times 10^{-6}}{d^4} = 0.52 + 0.173$$

$$\frac{3.75\times10^{-6}}{d^4} = 0.52+0.173-0.64$$

$$d^4 = \frac{3.75\times10^{-6}}{0.053}$$

$$d^4 = 7.075\times10^{-5}$$

$$\boxed{d = 0.092\ m}$$

2. Natural frequency of torsional vibration.

We know that.

Polar moment of Inertia, $J = \frac{\pi}{32}(d_1)^4$

$$J = \pi/_{32}\times(0.05)^4$$

$$\boxed{J = 0.614\times10^{-6}\ m^4}$$

Natural frequency of torsional vibration,

$$f_n = f_{nA}\ \text{(or)}\ f_{nB}$$

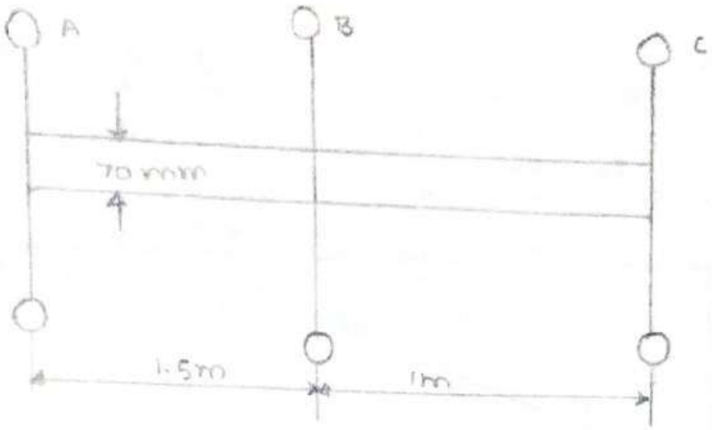
$$= 1/2\pi \times \sqrt{\frac{CJ}{\lambda_A I_A}}$$

$$f_n = 1/2\pi \times \sqrt{\frac{80\times10^9\times0.614\times10^{-6}}{0.52\times216}}$$

$$\boxed{f_n = 3.33\ Hz}$$

three rotor system.

A single cylinder oil engine drives directly a centrifugal pump. The rotating mass of the engine flywheel and the pump with the shaft equivalent to three rotor system as shown in figure.



1. The mass moment of inertia of the rotor ABC (or) 0.5, 0.3 and 0.09 kg.m².
Find the natural frequency of torsional vibration. The modulus of rigidity for the shaft material is 84 KN/mm².

Given.

$I_A = 0.15$ kg m²

$I_B = 0.3$ kg.m²

$I_C = 0.09$ kg m²

$C = 84$ KN/mm²

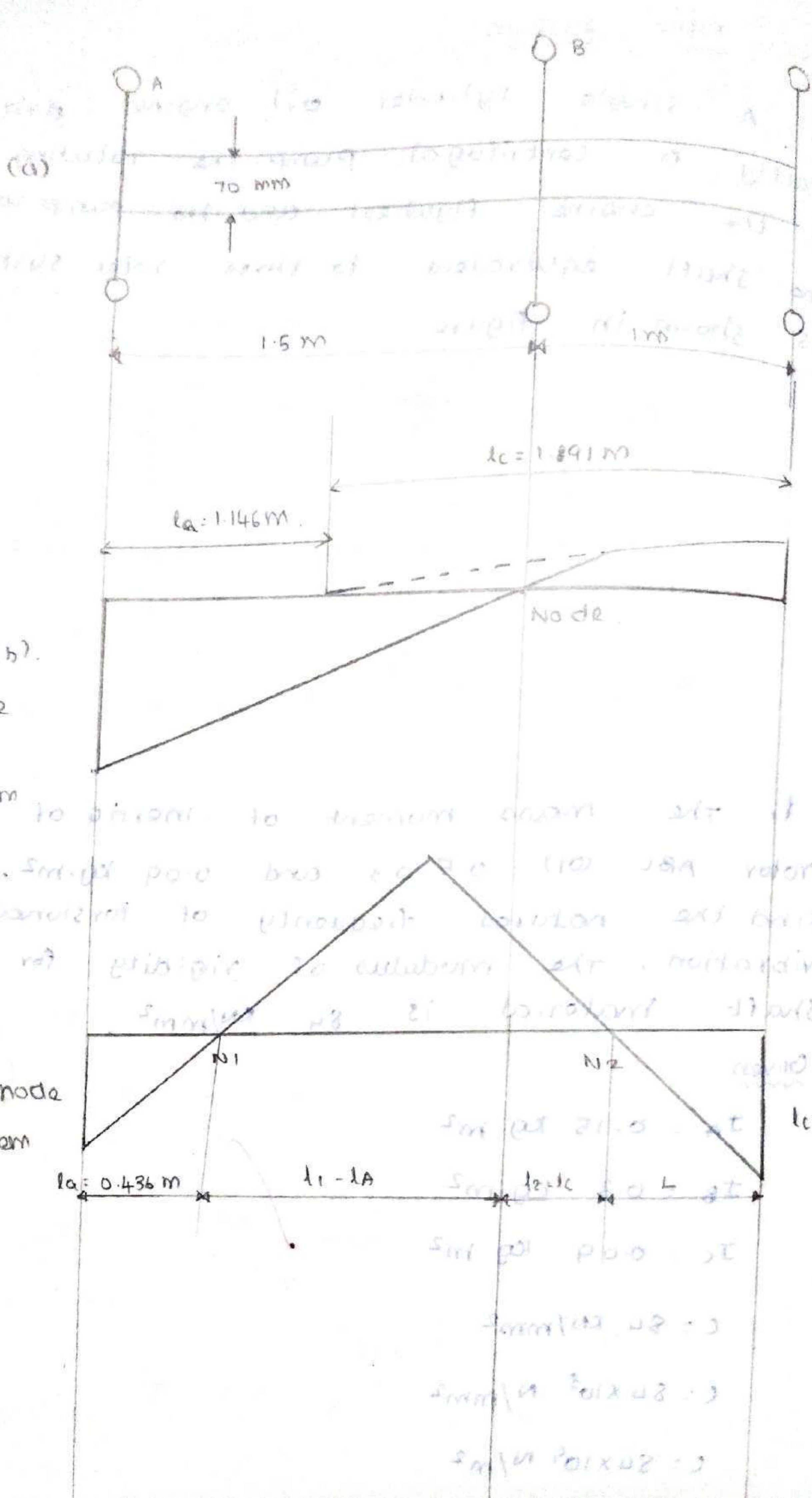
$C = 84 \times 10^3$ N/mm²

$C = 84 \times 10^9$ N/m²

$d = 70$ mm $= 0.07$ m

$l_1 = 1.5$ m

$l_2 = 1$ m



A
B
C
(a)
70 mm
1.5 m
1m
lc = 1.891 m
la: 1.146 m
Node
(b).
Single node system
(c)
Two node system
N1
N2
lc = 0.726m
la = 0.436 m
l1 - lA
l2=lc
L

W.K.T.

$$\lambda_A I_A = \lambda_C I_C$$

$$\lambda_A = \frac{\lambda_C I_C}{I_A}$$

$$\lambda_A = \lambda_C \times \frac{0.09}{0.15}$$

$$\lambda_A = 0.6\lambda_C$$

W.K.T.

$$\frac{1}{\lambda_C I_C} = \frac{1}{I_B}\left(\frac{1}{l_1 - \lambda_A} + \frac{1}{l_2 - \lambda_C}\right)$$

$$\frac{1}{\lambda_C \times 0.09} = \frac{1}{I_B}\left(\frac{1}{1.5 - \lambda_A} + \frac{1}{1 - \lambda_C}\right)$$

$$\frac{1}{\lambda_C \times 0.09} = \frac{1}{I_B}\left[\frac{1}{1.5 - (0.6\lambda_C)} + \frac{1}{1 - \lambda_C}\right]$$

$$[\because \lambda_A = 0.6\lambda_C]$$

$$\frac{1}{0.09\lambda_C} = \frac{1}{I_B}\left[\frac{1}{1.5 - (0.6\lambda_C)} + \frac{1}{1 - \lambda_C}\right]$$

$$\therefore \frac{1}{0.09\lambda_C} = \frac{1}{I_B}\left[\frac{1}{1.5 - 0.6\lambda_C} + \frac{1}{1 - \lambda_C}\right]$$

$$\frac{I_B}{0.09\lambda_C} = \left[\frac{1 - \lambda_C}{(1.5 - 0.6\lambda_C)(1 - \lambda_C)} + \frac{1.5 - 0.6\lambda_C}{(1.5 - 0.6\lambda_C)(1 - \lambda_C)}\right]$$

$$\frac{0.3}{0.09\lambda_C} = \frac{1 - \lambda_C + 1.5 - 0.6\lambda_C}{(1.5 - 0.6\lambda_C)(1 - \lambda_C)}$$

$$\frac{0.3}{l_c \times 0.09} = \frac{2.5 - 1.61 l_c}{1.5 - 2.1 l_c + 0.6 (l_c)^2}$$

$$0.3(1.5 - 2.1 l_c + 0.6 l_c^2) = (2.5 - 1.61 l_c)\, 0.09 l_c$$

$$0.45 - 0.63 l_c + 0.18 l_c^2 = 0.225 l_c - 0.144 l_c^2$$

$$0.324 l_c^2 - 0.855 l_c + 0.45 = 0$$

$l_c = 1.92$ m

(or)

$l_c = 0.73$ m

W.K.T.

$l_A = 0.6 \times l_c$

$l_A = 0.6 \times 1.92$

$l_A = 1.152$ m

$l_A = 0.6 \times 0.73$

$l_A = 0.438$ m

$l_A = 1.152$ m (or) 0.438 m

Here BC that when $l_c = 1.91$ m then $l_A = 1.152$ m. this gives the position of single node for $l_A = 1.152$ m shown in figure (B). the value of $l_c = 0.726$ m and corresponding value of $l_A = 0.438$ m gives the position of two nodes as shown in figure C.

polar moment of Inertia of the shaft,

$J = \pi/32 \times d^4$

$= \pi/32 \times (0.07)^4$

$\boxed{J = 2.36 \times 10^{-6}\ m^3}$

Natural Frequency of torsional vibration for a single node system fig(B)

$$f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}}$$

$$= \frac{1}{2\pi} \times \sqrt{\frac{84\times10^9 \times 2.36\times10^{-6}}{1.146 \times 0.15}}$$

$$\boxed{f_{n_1} = 171\ Hz}$$

Natural Frequency of torsional vibration for a two node system fig(C).

$$f_{n_2} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}}$$

$$= \frac{1}{2\pi} \times \sqrt{\frac{84\times10^9 \times 2.36\times10^{-6}}{0.4356 \times 0.15}}$$

$$\boxed{f_{n_2} = 277\ Hz}$$

Natural frequency of torsional vibration for a single node system fig (b)

$$f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A l_A}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{84\times10^9 \times 2.357\times10^{-6}}{0.15\times1.146}}$$

$$f_{n_1} = 170.81 \text{ Hz}$$

Natural frequency of torsional vibration for a two node system,

$$f_{n_2} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A I_A}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{84\times10^9 \times 2.357\times10^{-6}}{0.\cancel{35}\times0.4356}}$$

$$f_{n_2} = 277 \text{ Hz}$$

Result:

Natural frequencies of torsional vibrations of a 3 rotor system.

$$f_{n_1} = 171 \text{ Hz}$$

$$f_{n_2} = 277 \text{ Hz}.$$

4. FORCED VIBRATIONS.

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.

Three types of external forces applied are periodic forces (it includes harmonic and non-harmonic forces), impulsive type of force and random forces.

Forced vibrations with constant harmonic excitation:

(i) Equation of motion : $m * \frac{d^2x}{dt^2} + c.\frac{dx}{dt} + s.x = F_0 \sin\omega$

(ii) Complete solution,

$$x = X.e^{-\zeta\omega_n t}\sin(\omega_d t + \varphi_1) + \frac{F_0}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}}\sin(\omega t-$$

(iii) Amplitude (or the maximum displacement) of forced vibration is given by,

$$x_{max} = \frac{F_0}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}} = \frac{(F_0/s)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

where, r = Frequency ratio $= \omega/\omega_n$.

(iv) Phase lag for the displacement relative to the velocity vector is given by,

$$\varphi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

Magnification factor (or) dynamic magnifier:

The ratio of the maximum displacement of the forced (vibration (x_{max})) to the static deflection under the static force F_0 (x_0) is known as magnifying factor (M.F).

$$M.F = \frac{x_{max}}{x_0} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\frac{\omega_{max}}{\omega_n} = \sqrt{1-2\zeta^2} \quad \text{and} \quad \frac{\omega_d}{\omega_n} = \sqrt{1-\zeta^2}.$$

Forcing caused by unbalance:

(i) Equation of motion:

$$m\frac{d^2x}{dt^2} + c.\frac{dx}{dt} + cx = m_u \omega^2 e \sin\omega t.$$

(ii) Complete solution:

$$x = Xe^{-\zeta\omega_n t} . \sin(\omega_d t + \varphi_1) + \frac{m_u \omega^2 e}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}} \cos(\omega_t - \varphi)$$

(iii) Amplitude (or the maximum displacement) of forced vibration is given by

$$\frac{x_{max}}{\left(\frac{m_u \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

(iv) Phase angle, $\varphi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$.

Forced vibrations due to excitation of the support:

1. Absolute amplitude:

(i) Equation of motion: $m\frac{d^2x}{dt^2} + c.\frac{dx}{dt} + sx$

$$= Y\sqrt{s^2+(c\omega)^2} . \sin(\omega t + \alpha).$$

(ii) Amplitude of vibration:

$$\frac{x_{max}}{Y} = \frac{1+(2\zeta r)^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

where, r = Frequency ratio = ω/ω_n.

(iii) Phase lag: $\alpha = \tan^{-1}(2\zeta r)$

2. Relative amplitude:

(i) Equation of motion:

$$m\frac{d^2z}{dt^2} + c.\frac{dz}{dt} + sz = m\omega^2 Y \sin\omega t.$$

(ii) Steady state relative amplitude:

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

(iii) Dynamic load on isolators due to vibration,

$$F_{dyn} = \sqrt{s^2+(c\omega)^2}.$$

*. 2m

The process of reducing the vibrations of machines and hence reducing the transmitted force to the foundation using isolating materials is called vibration isolation.

*. 2m Transmissibility is defined as the ratio of the force transmitted (F_T) to the force applied (F_0) on the system.

$$\therefore \varepsilon = \frac{F_T}{F_0} = \frac{x_{max}}{y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}.$$

Part - B : (chapter - I)

Forced Vibrations with constant harmonic excitation

and

Magnification factor (or) Dynamic magnifier:

Pbms:

1) A mass of 50 kg is supported by an elastic structure of total stiffness 20 kN/m. The damping ratio of the system is 0.2. The simple harmonic disturbing force acts on the mass and at any time 't' seconds, the force is 60 sin 10 t (N). Find the amplitude of vibrations and the phase angle caused by

Gn:

$m = 50$ kg

$S = 20$ kN/m $= 20 \times 10^3$ N/m.

$\zeta = 0.2$.

$F = 60 \sin 10t$ (N).

Soln:

Since the periodic force, $F = F_0 \sin\omega t = 60 \sin 10t$

$\therefore$ Static force, $F_0 = 60$ N and angular velocity of the periodic disturbing force, $\omega = 10$ rad/se

W.K.T,

Natural frequency of angular vibration,

$$\omega_n = \sqrt{S/m}.$$

$$= \sqrt{\frac{20 \times 10^3}{50}}$$

$$\omega_n = 20 \text{ rad/s}.$$

$\therefore$ Frequency ratio, $r = \frac{\omega}{\omega_n} = \frac{10}{20} = 0.5$.

(i) Amplidude of forced vibrations: (x_{max}):

W.K.T,

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{60/20 \times 10^3}{}$$

$$= \frac{3 \times 10^{-3}}{0.7763}$$

$$= 3.86 \times 10^{-3} \text{ m.}$$

$$= 3.86 \text{ mm}$$

(ii) Phase angle caused by damping (φ):

W.K.T,

$$\varphi = \tan^{-1}\left[\frac{2 \zeta r}{1 - r^2}\right].$$

$$= \tan^{-1}\left[\frac{2 \times 0.2 \times 0.5}{1 - 0.5^2}\right].$$

$$\varphi = 14.93^{\circ}.$$

2) A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to 1/10 of the initial value in four complete oscillations. With a periodic force of 150 cos 50 t N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations.

What is it's value of resonance?

Gn:

$m = 10$ kg

$S = 10$ N/mm $= 10 \times 10^3$ N/m.

$$X_4 = \frac{1}{10} X_0$$

$$\frac{X_0}{X_4} = 10$$

$F = 150 \cos 50 t$.

$F = F_0 \cos \omega t$.

$\therefore F_0 = 150$ N; $\omega = 50$ rad/s.

Soln:

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{10 \times 10^3}{10}} = 31.62 \text{ rad/s.}$$

$$\text{Frequency ratio, } r = \frac{\omega}{\omega_n} = \frac{50}{31.62} = 1.581.$$

(i) Amplitude of the forced vibration:

Let, $\zeta \longrightarrow$ Damping factor. (damping ratio)

For 'n' cycles W.K.T, The lograthmic decrement,

$$\delta = \frac{1}{n} \ln\left[\frac{x_0}{x_n}\right].$$

$$= \frac{1}{4} \ln\left[10\right]$$

$$\delta = 0.5757$$

Also, W.K.T,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$0.5756 = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$(0.5756)^2(1-\zeta^2) = 4\pi^2\zeta^2$$

$$0.331 - 0.331\zeta^2 = 39.48\zeta^2$$

$$0.331 = (39.48 + 0.331)\zeta^2$$

$$\therefore \zeta = 0.0912$$

W.K.T, $$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$= \frac{150/10\times10^3}{\sqrt{(1-1.581^2)^2 + [2(0.0912)(1.581)]^2}}$$

$$= 9.823 \times 10^{-3}\ m$$

$x_{max} = 9.823$ mm.

(iii) Amplitude of forced vibration at resonance:

At resonance, $(r = 1)$

$$x_{max} = \frac{F_0}{2\zeta(s)}$$

$$= \frac{150}{2(0.0912)(10^4)}$$

$$= 0.08224\ m$$

$x_{max} = 82.24$ mm

Result:

(i) Amplitude of forced vibration = 9.823 mm.

(ii) " " " " at resonance = 82.24 mm

3) A harmonic exciting force of 25 N is acting on a machine part which is having a mass of 2 kg and is vibrating in a viscous medium. The exciting force causes a resonant amplitude of 12.5 mm with a period of 0.20 sec. Determine the damping coefficient.

Gn:

$F_0 = 25$ N

$m = 2$ kg

x_{max} at resonance $= 12.5$ mm.

$t_p = 0.20$ sec.

Soln:

W.K.T, $$\omega_n = \frac{2\pi}{t_p}$$

$$= \frac{2\pi}{0.2}$$

$\omega_n = 31.416$ rad/s.

and $\omega_n = \sqrt{S/m}$

$$31.46 = \sqrt{\frac{S}{2}}$$

$\Rightarrow S = 1979.46$ N/m $= 1979.4 \times 10^{-3}$ N/m

At resonance maximum amplitude of vibration,

$$x_{max} = \frac{F_0}{2\zeta S}$$

$$12.5 = \frac{25}{2 \times \zeta \times 1979.46 \times 10^{-3}}$$

$\zeta = 0.5051$.

W.K.T,

$$C = 2 m \omega_n \zeta$$

$$= 2 \times 2 \times 31.416 \times 0.5051$$

$C = 63.48$ N/m/s

4) In the above problem, if the system is excited by a harmonic force of frequency 4 Hz. find the increase in amplitude of forced vibration when damper is removed.

Gn:

$f = 4$ Hz

Soln:

W.K.T, $\omega_n = 2\pi f$ $\quad \left[\because f_n = \frac{1}{2\pi}\omega_n \right]$.

$\omega_n = 2\pi \times 4$.

$\omega_n = 25.133$ rad/sec.

Amplitude with damper:

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \text{——— (i)}$$

where, $r = \frac{\omega}{\omega_n}$

$$= \frac{25.133}{31.416}$$

$r = 0.8$.

$$\therefore x_{max} = \frac{25/1979.46}{\sqrt{(1-0.8^2)^2 + (2 \times 0.5051 \times 0.8)^2}}$$

$= 0.01428$ m.

$x_{max} = 14.28$ mm.

Amplitude without damper:

W.K.T, the amplitude of vibration without damper, $\zeta = 0$ in ①

$$\therefore x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2}}$$

$$= \frac{25/1979.46}{\sqrt{(1-0.8^2)^2}}$$

$$= 0.03518 m$$

$$x_{max} = 35.18 mm.$$

Increase in amplitude of vibration:

Increase in amplitude of vibration = Amplitude without damper - Amplitude with damper.

$$= 35.18 - 14.28$$

$$= 20.9 mm.$$

5) A mass of 500 kg is mounted on supports having a total stiffness of 100 kN/m and which provides viscous damping, damping ratio being 0.4. The mass is constrained to move vertically and is subjected to a vertical disturbing force of type $F_0 \sin \omega t$. Determine the frequency at which resonance will occur and the maximum allowable value of F_0 if the amplitude at resonance is to be restricted to 5mm.

Gn:

$m = 100$ kg

$s = 100 \times 10^{3}$ N/m.

$\zeta = 0.4$

$x_{max} = 5 mm = 5 \times 10^{-3}$ m.

Soln:

Frequency of at which resonance will occur:

The resonance will occur at $\omega = \omega_n$.

$$\omega = \omega_n = \sqrt{s/m} = \sqrt{\frac{100 \times 10^3}{500}} = 14.14 \text{ rad/s}$$

Frequency at which resonance will occur is given by,

$$f = f_n = \frac{\omega}{2\pi}$$

$$= \frac{14.14}{2\pi}$$

$$f = 2.25 \text{ Hz} \quad \text{Ans (i)}$$

Maximum allowable value of F_0:

W.K.T, Maximum displacement of forced vibration at resonance,

$$x_{max} = \frac{F_0}{2\zeta s}.$$

Result :

(i) Frequency at which resonance occurs = 2.25 Hz

(ii) Maximum allowable value of F_0 = 400 N.

6) The damped natural frequency of a system as obtained from a free vibration test is 7.7 Hz. During the forced vibration test, with constant exciting force on the same system, the maximum amplitude of vibration is found to be at 7.5 Hz. Find the damping factor for the system and it's natural frequency.

Gn:

$f_d = 7.7$ Hz

$f_{max} = 7.5$ Hz.

Soln:

(i) Damping factor: (ζ)

W.K.T, $\omega_d = f_d \times 2\pi = 48.38$ rad/s.

$\omega_{max} = f_{max} \times 2\pi$

$\omega_{max} = 47.12$ rad/s.

W.K.T,

$$\frac{\omega_d}{\omega_n} = \sqrt{1-\zeta^2}$$

$$\frac{48.38}{\omega_n} = \sqrt{1-\zeta^2} \quad \text{——} \; (1)$$

$$\frac{47.12}{\omega_n} = \sqrt{1-2\zeta^2} \quad \text{——} \; (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{48.38}{47.12} = \frac{\sqrt{1-\zeta^2}}{\sqrt{1-2\zeta^2}}$$

$$\left(\frac{48.38}{47.12}\right)^2 = \frac{1-\zeta^2}{1-2\zeta^2}.$$

$$1.054 = \frac{1-\zeta^2}{1-2\zeta^2}.$$

$$(1.054)(1-2\zeta^2) = 1-\zeta^2.$$

$$1.054 - 2.108\zeta^2 = 1 - \zeta^2.$$

$$1.054 - 1 = -\zeta^2 + 2.108\zeta^2.$$

$$1.108\zeta^2 = 0.054$$

$$\therefore \zeta = 0.221 \rightarrow \text{Ans}$$

(ii) Natural frequency of the system : (f_n) :

$$(1) \Rightarrow \frac{48.38}{\omega_n} = \sqrt{1-(0.221)^2}.$$

$$\omega_n = \frac{48.38}{0.975}$$

$$\omega_n = 49.6 \text{ rad/s}.$$

$\therefore$ natural frequency, $f_n = \frac{\omega_n}{2\pi}$.

$$= \frac{49.6}{2\pi}$$

Result :

(i) Damping factor, $\xi = 0.221$.

(ii) Natural frequency, $f_n = 7.895$ Hz.

7) A spring mass system is excited by a force $F \sin \omega t$. On measuring the amplitude of vibration is found to be 12mm at resonance. However at a frequency 0.8 times the natural frequency the amplitude reduces to 8 mm. Determine the damping ratio of the system.

Gn :

x_{max} at resonance $= 12\,mm = 0.012\,m$.

$\omega = 0.8\,\omega_n$.

$\frac{\omega}{\omega_n} = 0.8$

$\Rightarrow r = 0.8$

$x_{max} = 8\,mm = 0.008\,m$.

Soln :

At resonance,

$$x_{max} = \frac{F_0}{2\xi s}$$

$$x_{max} = \frac{F_0/s}{2\xi}$$

$$0.012 = \frac{F_0/s}{2\xi} \quad \text{—} \; ①.$$

Amplitude of forced vibration,

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$0.008 = \frac{F_0/s}{\sqrt{(1-0.8^2)^2 + [(2\xi)(0.8)]^2}}$$

$$0.008 = \frac{F_0/s}{\sqrt{0.1296 + 2.56\xi^2}} \quad \text{—} \; ②.$$

$$\frac{①}{②} \Rightarrow \frac{0.012}{0.008} = \frac{\frac{F_0/s}{2\xi}}{\frac{F_0/s}{\sqrt{0.1296+2.56\xi^2}}}$$

$$1.5 = \frac{\sqrt{0.1296 + 2.56\xi^2}}{2\xi}$$

$$9\xi^2 = 0.1296 + 2.56\xi^2.$$

$$6.44\xi^2 = 0.1296$$

$$\xi = 0.142$$

Result :

Damping ratio, $\xi = 0.142$.

chapter - 2 :

Forcing caused by unbalance:

Almost in all rotating and reciprocating machinary like an electrical motor, a turbine, an IC engine, etc... have some amount of unbalanced force left in them even after correcting their unbalance on precision balancing machines. This small unbalanced force produces the exciting force in a machine.

Let,

$m \rightarrow$ Vibrating mass.

$m_u \rightarrow$ unbalanced mass.

$e \rightarrow$ eccentricity $= \frac{\text{stroke}}{2}$

$S \rightarrow$ spring stiffness.

$C \rightarrow$ damping coefficient.

1) A single cylinder vertical petrol engine of total mass of 200 kg is mounted upon the steel frame. The vertical static deflection of the frame is 2.4 mm due to the weight of the engine. The mass of the reciprocating parts is 9 kg and its stroke of the piston is 160 mm with simple harmonic motion. If

to dampen the vibrations, calculate at steady state.

(i) The amplitude of forced vibrations at 500 rpm engine speed.

(ii) The speed of driving shaft at which resonance will occur.

Gn:

$m = 200$ kg.

$\delta = 2.4$ mm $= 0.0024$ m.

$m_u = 9$ kg

$L = 160$ mm $= 0.16$ m.

$C = 1$ N/mm/s $= 1000$ N/m/s.

$N = 500$ rpm.

Soln:

$$\omega = \frac{2\pi N}{60} = 52.36 \text{ rad/s}.$$

$$\omega_n = \sqrt{S/m} = \sqrt{g/\delta}$$

$$\omega_n = \sqrt{\frac{9.81}{0.0024}}$$

$$\omega_n = 63.93 \text{ rad/s}.$$

- Frequency ratio,

$$\therefore r = \frac{\omega}{\omega_n} = \frac{52.36}{63.93} = 0.819,$$

$$\text{Damping factor, } \zeta = \frac{c}{2m\omega_n} \quad \left[\because C_c = 2m\omega_n\right].$$

$$= \frac{1000}{2 \times 200 \times 63.93}$$

$$\xi = 0.0391$$

Eccentricity, $e = \frac{\text{stroke}}{2}$

$$= \frac{0.16}{2}$$

$$e = 0.08 \text{ m}.$$

(i) Amplitude of steady state forced vibrations:

We know that,

the amplitude of forced vibration caused by unbalance

$$\frac{x_{max}}{\left(\frac{m_u \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{x_{max}}{\left(\frac{9(0.08)}{200}\right)} = \frac{(0.819)^2}{\sqrt{(1-0.819^2)^2 + [2(0.0391)(0.819)]^2}}$$

$$x_{max} = \frac{3.6 \times 10^{-3} \times (0.819)^2}{0.335}$$

$$\therefore x_{max} = 7.208 \times 10^{-3} \text{ m}.$$

$$\Rightarrow x_{max} = 7.208 \text{ mm}. \quad \text{Ans(i)}$$

(ii) Speed of the driving shaft at which resonance will occur:

$\omega = \omega_n$ Angular speed at which resonance occurs, $\omega_n = 63.93$ rad/s.

Also, $\omega_n = \frac{2\pi N}{60}$

$$63.93 = \frac{2\pi N}{60}$$

$$\therefore N = 610.5 \text{ rpm}. \quad \text{Ans(ii)}$$

2) A vertical single stage air compressor having a mass of 500 kg is mounted on springs having stiffness of 1.96×10^5 N/m and dashpots with a damping factor of 0.2m. The rotating parts are completely balanced and the equivalent reciprocating parts of 20 kg weight. The stroke is 0.2 m. Determi[ne] the dynamic amplitude of vertical motion and the pha[se] difference between the motion and the excitation force if the compressor is operated at 200 rpm.

Gn:

$S = 1.96 \times 10^5$ N/m.

$\xi = 0.2$ m.

$N = 200$ rpm

stroke = 0.2m.

Soln:

$$\omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

$$\omega_n = \sqrt{S/m} = \sqrt{\frac{1.96\times 10^5}{500}} = 19.8 \text{ rad/s}.$$

$\therefore$ Frequency ratio, $r = \frac{\omega}{\omega_n} = 1.057$.

$$e = \frac{\text{stroke}}{2} = \frac{0.2}{2} = 0.1\text{ m}$$

i) Dynamic amplitude of vertical motion:

$$\frac{x_{max}}{\left(\frac{m_a \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\frac{x_{max}}{\left(\frac{20\times 0.1}{500}\right)} = \frac{(1.057)^2}{\sqrt{(1-1.057^2)^2 + [2(0.2)(1.057)]^2}}$$

$$x_{max} = \frac{4.468\times 10^{-3}}{0.439}$$

$$x_{max} = 0.0102\text{ m}$$

$$x_{max} = 10.2\text{ mm}.$$

(ii) Phase difference between the motion and the excitation force: (φ)

$$\varphi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

$$= \tan^{-1}\left(\frac{2\times 0.2\times 1.057}{1-(1.057)^2}\right).$$

$$\varphi = -74.5^\circ$$

(or)

$$\varphi = -74.5 + 180 \quad [\because \tan(180^\circ + \theta) = \tan\theta].$$

$$\varphi = 105.64^\circ.$$

Chapter - 3:

Forced Vibrations due to excitation of the support:
(support motion)

In many application (ie) external force is applied through the base of ~~the~~ or the support instead of being applied to the mass.

Let the support or the base is excited by a regular sinusoidal motion, $y = Y\sin\omega t$.

1) The support of a spring mass system is vibrating with an amplitude of 6 mm and a frequency of 20 Hz. If the mass is 1.1 kg and the spring has a stiffness of 2000 N/m, determine the amplitude of vibration of the mass as when $\zeta = 0$. What amplitude

Gn:

$Y = 6\ mm = 0.006\ m.$

$f = 20\ Hz$

$m = 1.1\ kg$

$S = 2000\ N/m.$

Soln:

$\omega = 2\pi f = 2\pi \times 20 = 125.66\ rad/s$

Natural frequency, $\omega_n = \sqrt{S/m}$

$$= \sqrt{\frac{2000}{1.1}}$$

$$\omega_n = 42.64\ rad/s.$$

Frequency ratio, $r = \frac{\omega}{\omega_n} = \frac{125.66}{42.64}$

$\therefore r = 2.946$

i) Amplitude of vibration when $\zeta = 0$:

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$x_{max} = \frac{6\sqrt{1-\cancel{(2(0)r)^2}}}{\sqrt{[1-(2.946)^2]^2 + \cancel{[2(0)r]^2}}}$$

Subs $\zeta = 0$ bcz spring mass system is located without damper.

$$\Rightarrow \frac{x_{max}}{Y} = \frac{1}{1-r^2}$$

Since $r > 1$, $\frac{x_{max}}{Y} = \frac{1}{r^2 - 1}$

$$x_{max} = \frac{0.006}{(2.946)^2 - 1} = 7.807 \times 10^{-4}$$

$$x_{max} = 0.781\ mm.$$

(ii) Amplitude of vibration at $\zeta = 0.25$:

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$x_{max} = \frac{0.006\sqrt{1+[2(0.25)(2.946)]}}{\sqrt{(1-(2.946^2))^2 + (2(0.25)(2.946))}}$$

$$= \frac{0.006 \times 1.78}{7.818}$$

$$x_{max} = 1.366 \times 10^{-3}\ m.$$

$$\therefore x_{max} = 1.366\ mm.$$

2) A computer moniter set of 18 kg mass must be isolated from a machine vibrating with an amplitude 0.06 mm at 520 rpm. The set is mounted on four isolators, each having a spring scale of 31000 N/m and damping coefficient of 400 N/m/s.

(i) What is the amplitude of vibration of the computer monitor?

(ii) What is the dynamic load on each isolator due to vibration?

Gn:

$m = 18$ kg.

$Y = 0.06\,mm = 0.06 \times 10^{-3}$ m.

$N = 520$ rpm.

$S = 31000$ N/m.

$c = 400$ N/m/s.

no. of isolators = 4.

Soln:

Since there are four isolator

∴ The equivalent stiffness (S_{eq}) and equivalent damping coefficient (C_{eq}).

$$S_{eq} = 4 \times S = 4 \times 31000 = 124000\ N/m$$

and, $C_{eq} = 4 \times C = 4 \times 400 = 1600$ N/m/s.

Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi(520)}{60} = 54.45$ rad/s

Natural frequency, $\omega_n = \sqrt{S/m} = \sqrt{\frac{124000}{18}} = 83$ rad/s

Frequency ratio, $r = \frac{\omega}{\omega_n} = \frac{54.45}{83} = 0.656$.

Damping factor, $\xi = \frac{c}{2m\omega_n}$

$$= \frac{1600}{2 \times 18 \times 83}$$

$$\xi = 0.535$$

(i) Amplitude of the computer monitor: (x_{max}):

$$\frac{x_{max}}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$x_{max} = \frac{0.06 \times 10^{-3}\sqrt{1+(2 \times 0.535 \times 0.656)}}{\sqrt{(1-0.656^2)^2+[2(0.535)(0.656)]}}$$

$$= \frac{7.3305 \times 10^{-5}}{0.904}$$

$$= 8.12 \times 10^{-5}\ m$$

$$x_{max} = 0.0812\ mm.$$

(ii) Dynamic load on each isolator:

The dynamic load is given by.

$$F_{dyn} = z\sqrt{(c\omega)^2 + s^2}$$

The value of relative amplitude 'z' is given by,

$$Z = \quad r^2$$

$$Z = \frac{0.06 \times 10^{-3} \times (0.656)^2}{\sqrt{(1-0.656^2)^2 + (2 \times 0.535 \times 0.656)^2}}$$

$$= \frac{2.582 \times 10^{-5}}{0.904}$$

$$Z = 2.856 \times 10^{-5} \text{ m}$$

$$Z = 0.0285 \text{ mm}$$

$$\therefore F_{dyn} = (2.85 \times 10^{-5}) \sqrt{[(1600)(54.45)]^2 + (184000)^2}$$

$$= 4.326$$

Hence the dynamic load on each isolator

$$= \frac{4.326}{4}$$

$$= 1.08 \text{ N}.$$

Chapter - 4:

Vibration Isolation:

The process of reducing the vibrations of machines and hence reducing the transmitted force to the foundation using vibration isolating materials is called vibration isolation.

Isolating Materials:

1) Rubber
2) Felt
3) Cork
4) Metallic springs.

Types of isolation:

1) Isolation of forces.
2) Isolation of motions.

Isolation of forces:

Vibrations produced in unbalanced machines should be isolated from the foundati This type of isolation is known as a forced isolation.

For eg: A product such as computer monitor which is not of itself, a vibration generator, may receive from another source. To overcome these vibrations, forced isolation is to be done.

Isolation of motions:

The unbalanced machines are isolated from their foundations so that there should not be any damage either to the machines or the foundations during motions. This type of isolation is known as motion isolation.

* Transmissibility:

Transmissibility is a measure of effectiveness of the vibration isolating materials.

Force transmissibility (or) Isolating factor:

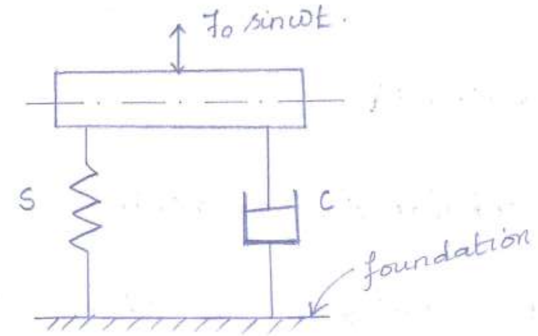
It is defined as the ratio of the force transmitted (F_T) (to the foundation) to the ~~force~~ force applied (F_0) on the system. This ratio is known as isolating factor and is denoted by 'ε'.

Mathematically,

$$\text{Force transmissibility} = \frac{F_T \text{ (to the foundation)}}{F_0 \text{ (on the system)}}$$

$$\varepsilon = \frac{F_T}{F_0}$$

Derivation to find force transmissibility:



Consider a mass 'm' supported on the foundation by means of an isolator as shown in figure. Let the mass is excited by a simple harm[onic] force, $F = F_0 \sin\omega t$. The force transmitted to the foundation consists of the following two cases

1) Spring force (or) elastic force $= S \cdot x_{max}$
2) Damping force $= c\omega x_{max}$.

The resultant force F_T is given by,

$$F_T = \sqrt{(S \cdot x_{max})^2 + (c\omega x_{max})^2}$$

$$= \sqrt{x_{max}^2 (S^2 + c^2\omega^2)}$$

$$F_T = x_{max}\left(\sqrt{S^2 + c^2\omega^2}\right) \quad \text{--- (1)}$$

Transmissibility ratio, $\varepsilon = \frac{F_T}{F_0}$.

$$\varepsilon = \frac{x_{max}\left(\sqrt{S^2 + c^2\omega^2}\right)}{F_0} \quad \text{--- (2)}$$

W.K.T,

$$x_{max} = \frac{F_0}{\sqrt{(S - m\omega^2)^2 + (c\omega)^2}} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \quad \text{--- (3)}$$

Case(i):

Substitute eqn (3) in eqn (2), (ie) Substitute x_{max} value in (2),

$\therefore$ eqn (2) becomes,

$$\varepsilon = \frac{F_0}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}} \times \frac{\sqrt{s^2+(c\omega)^2}}{F_0}$$

$$\varepsilon = \frac{\sqrt{s^2+(c\omega)^2}}{\sqrt{(s-m\omega^2)^2+(c\omega)^2}} \quad \text{—— (4)}$$

<u>Case (ii)</u> :

Subs eqn (3) in eqn (2),

∴ eqn (2) becomes,

$$\varepsilon = \frac{F_0/s}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \times \frac{\sqrt{s^2+c^2\omega^2}}{F_0}$$

$$\varepsilon = \frac{\sqrt{s^2+c^2\omega^2}}{s\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\varepsilon = \frac{\sqrt{\dfrac{s^2+(c\omega)^2}{s^2}}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{\sqrt{\dfrac{s^2}{s^2}+\dfrac{(c\omega)^2}{s^2}}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{c\omega}{s}\right)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \quad \text{—— (5)}$$

W.K.T, $c = 2\zeta m\omega_n$ —— (6)

Now, $\omega_n = \sqrt{s/m}$

$$s/m = \omega_n^2$$

$$m = s/\omega_n^2 \quad \text{—— (7)}$$

Subs eqn (7) in eqn (6),

$$\text{eqn (6)} \Longrightarrow c = 2\zeta \frac{s}{\omega_n^2} \times \omega_n$$

$$\therefore c = \frac{2\zeta s}{\omega_n} \quad \text{—— (8)}$$

Now substitute eqn (8) in eqn (5),

(ie) Subs 'c' value in eqn (5)

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2\zeta s}{s\omega_n}\times\omega\right)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\boxed{\varepsilon = \frac{F_T}{F_0} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \quad \text{—— (9)}} \quad \left[\because r = \omega/\omega_n\right]$$

Equation (9) represents 'equation of force transmissibility'.

Note :

(i) At resonance, $\omega = \omega_n$; $r = 1$, Transmissibility,

ii) When no damper is used, $\xi = 0$,

$\therefore$ Transmissibility, $\varepsilon = \pm \frac{1}{1-r^2}$.

iii) When $(r = \omega/\omega_n) < 1$, then $\varepsilon = \frac{1}{1-r^2}$.

$r > 1$, then, $\varepsilon = \frac{1}{r^2-1}$.

2m:

Phase lag:

Let, $\alpha \rightarrow$ (Phase angle of the transmitted force (F_t) with respect to displacement)

$\phi \rightarrow$ (Phase angle of the impressed force (F_o) with respect to displacement).

$\phi - \alpha \rightarrow$ Phase lag of the transmitted force (F_t) with respect the impressed force (F_o)

$r \rightarrow$ Frequency ratio $= \omega/\omega_n$.

Now, $\alpha = \tan^{-1}(2\xi r)$.

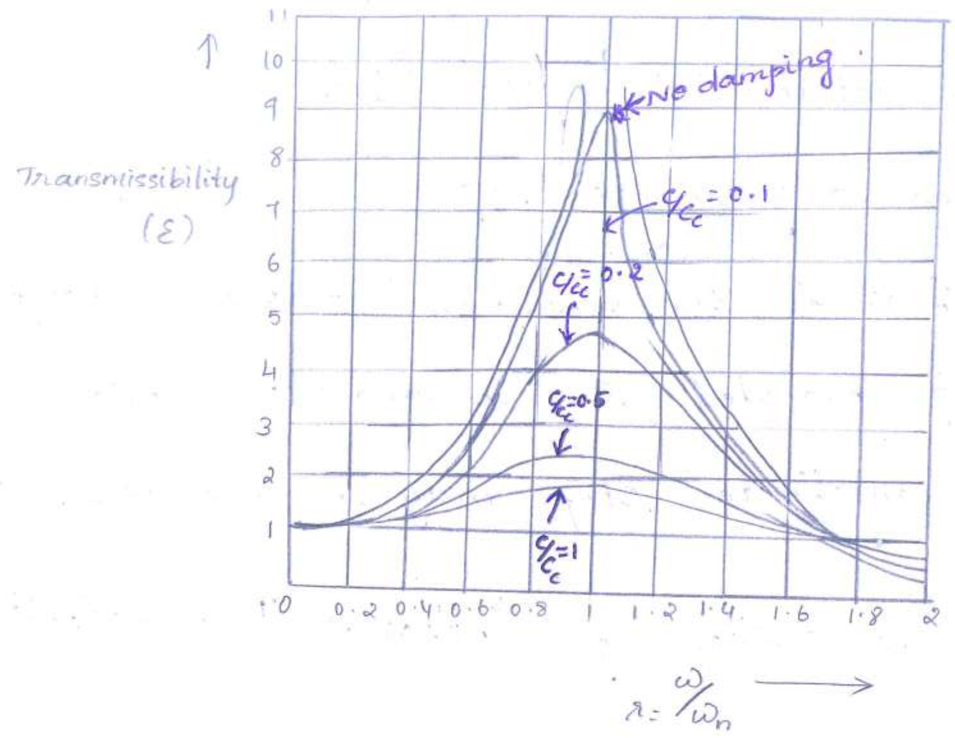
$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$\therefore \text{Phase lag, } \phi - \alpha = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right) - \tan^{-1}(2\xi r)$$

★ 6m:

Transmissibility Vs Frequency ratio:

The figure is the graph for different values of damping factor c/c_c to show the variation



of transmissibility ratio ε against the ratio ω/ω_n

$F_t = F_o$ 1) When $\omega/\omega_n = \sqrt{2}$, then all the curves pass through the point $\varepsilon = 1$ for all values of damping factor c/c_c.

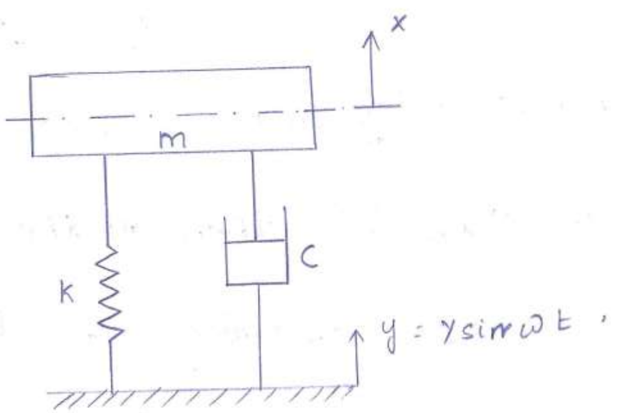
$F_t > F_o$ 2) When $\omega/\omega_n < \sqrt{2}$, then $\varepsilon > 1$ for all values of damping factor c/c_c. This means that the force transmitted to the foundation through elastic support is greater than the force applied.

$F_t < F_o$ 3) When $\omega/\omega_n > \sqrt{2}$, then $\varepsilon < 1$ for all values of damping factor c/c_c. This shows that the force transmitted through elastic support is less than the applied force. Thus vibration isolation is possible only in the range of $\omega/\omega_n > \sqrt{2}$.

$F_t = D$ 4) When $r = 1$, then the transmitted force is

We also see from the graph that the damping is detrimental beyond $\omega/\omega_n > \sqrt{2}$ and advantageous only in the region $\omega/\omega_n < \sqrt{2}$. It is concluded that for the vibration isolation, dampers need not be provided but in order to limit resonance amplitude, stops may be provided.

Motion Transmissibility (or) Amplitude Transmissibility:



Amplitude transmissibility is defined as the ratio of absolute amplitude of the mass (x_{max}) to the base or support amplitude (y)

$$\text{Amplitude Transmissibility} = \frac{\text{Absolute amplitude of the mass}}{\text{Amplitude of the base}}$$

shown in fig.

Let $x_{max} \rightarrow$ Maximum / absolute amplitude of mass 'm'.

$y \rightarrow$ Amplitude of the base excitation.

Then, the amplitude transmissibility is given by,

$$\varepsilon_A = \frac{x_{max}}{Y} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

1) A machine 100 kg has a 20 kg rotar with 0.5 mm eccentricity. The mounting springs have $S = 85 \times 10^3$ N/m. The damping ratio is 0.02. The operating speed is 600 rpm and the unit is constrained to move vertically. Find

(i) Dynamic amplitude of the machine

(ii) The force transmitted to the supports.

Soln:

Gn:

$S = 85 \times 10^3$ N/m :

Soln:

Angular velocity of unbalanced force,

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 600}{60}$$

$$\therefore \omega = 62.83 \text{ rad/s.}$$

Natural circular frequency, $\omega_n = \sqrt{S/m}$

$$= \sqrt{\frac{85 \times 10^3}{100}}$$

$$\omega_n = 29.15 \text{ rad/s.}$$

$$\therefore \text{frequency ratio } r = \frac{\omega}{\omega_n} = \frac{62.83}{29.15} = 2.155.$$

unbalanced exciting force, $F_0 = m_u \omega^2 e$.

$$= 20\,(62.83)^2\,(0.5 \times 10^{-3})$$

$$F_0 = 39.48 \text{ N.}$$

(i) Dynamic Amplitude of the machine (x_{max}):

$$x_{max} = \frac{F_0/S}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$= \frac{39.48/85 \times 10^3}{\sqrt{(1-2.155^2)^2 + [2\,(0.02 \times 2.155)]^2}}$$

$$= \frac{4.645 \times 10^{-4}}{}$$

ii) Force transmitted to the supports: (F_T):

W.K.T,

$$\varepsilon = \frac{F_T}{F_0} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{F_T}{39.48} = \frac{\sqrt{1 + [2\,(0.02 \times 2.155)]^2}}{\sqrt{(1-(2.155)^2)^2 + (2 \times 0.02 \times 2.155)^2}}$$

$$F_T = \frac{1.004 \times 39.48}{3.65}$$

$$F_T = 10.86 \text{ N.}$$

Result:

(i) Dynamic amplitude of the machine, $x_{max} = 0.127$

(ii) Force transmitted to the supports, $F_T = 10.86$ N

2) A single cylinder engine has an out of balance force of 500 N at an engine speed of 300 rpm. The total mass of the engine is 150 kg and it is carried on a set of springs of total stiffness 300 N/cm.

i) Find the amplitude of the steady motion (no damper) of the mass and the maximum oscillating force

(ii) If a viscous damping is interposed between the mass and foundation, the damping force being 1000 N at 1 m/s of velocity. Find the amplitude of the forced damped oscillation of the mass and it's angle of lag with disturbing force.

Gn:

$F_0 = 500$ N

$N = 300$ rpm

$m = 150$ kg

$S = 300$ N/cm $= 300 \times 10^2$ N/m.

$C = 1000$ N at 1 m/s.

$\Rightarrow c = 1000$ N/m/s.

Soln:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60}$$

$\omega = 31.42$ rad/sec.

Natural circular frequency, $\omega_n = \sqrt{S/m}$

$$= \sqrt{\frac{300 \times 10^2}{150}}$$

$\omega_n = 14.14$ rad/s.

$\therefore$ frequency ratio, $r = \frac{\omega}{\omega_n}$

$$= \frac{31.42}{14.14}$$

$r = 2.222$.

(i) x_{max} and F_T when there in no damper:

We know that amplitude of forced vibration with no damper, $\xi = 0$,

$$x_{max} = \frac{F_0/s}{1 - r^2}$$

when $r > 1$, $x_{max} = \frac{F_0/s}{r^2 - 1}$

$$= \frac{500/300 \times 10^2}{2.222^2 - 1}$$

$x_{max} = 4.23 \times 10^{-3}$ m.

$x_{max} = 4.23$ mm.

Also force transmitted to the foundation is given by,

$$\varepsilon = \frac{F_T}{F_0} = \pm \frac{1}{1 - r^2} \quad [\because \text{When } \xi = 0].$$

when $r > 1$, $\varepsilon = \frac{F_T}{F_0} = \frac{1}{r^2 - 1}$

$$\therefore \frac{F_T}{500} = \frac{1}{2.222^2 - 1}$$

$F_T = 126.99$

ii) x_{max} and φ when there is a damper:

$$c = 1000 \text{ N/m/s}.$$

We know that, $\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n}$.

$c = 2m\omega_n \xi$

$\xi = \frac{c}{2m\omega_n}$

$$= \frac{1000}{2 \times 150 \times 14.14}$$

$$\xi = 0.236.$$

The amplitude of forced damped vibration is given by,

$$x_{max} = \frac{F_0/s}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$= \frac{500/300 \times 10^2}{\sqrt{(1-2.222^2)^2 + (2 \times 0.236 \times 2.222)^2}}$$

$$= \frac{500}{122237.22}$$

$$= 4.09 \times 10^{-3} \text{ m}.$$

$$x_{max} = 4.09 \text{ mm}.$$

The angle of lag with respect to exciting force, $\varphi = \tan^{-1}\left[\frac{2\xi r}{1-r^2}\right]$.

$$= \tan^{-1}\left[\frac{2 \times 0.236 \times 2.222}{1 - 2.222^2}\right].$$

$$\varphi = -14.92^\circ \Rightarrow \varphi = -14.92 + 180.$$

$$\varphi = 165.08^\circ.$$

2) The machine of mass 75 kg is mounted on spring stiffness 12×10^5 N/m and with an assumed damping factor of 0.2. A piston within the machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm and a speed of 3000 cycles/minute. Assuming the motion to be simple harmonic. Find (i) Amplitude of motion of the machine (ii) Phase angle with respect to the exciting force (iii) force transmitted to the foundation. (iv) Phase ~~of~~ angle of tramitted force with respect to exciting force (v) The phase lag of the transmitted force with respect to applied force.

Gn:

$m = 75$ kg

$s = 12 \times 10^5$ N/m.

$m_u = 2$ kg

$\xi = 0.2$.

stroke = 80 mm.

$N = 3000$ cycles/min.

Soln:

Angular velocity of unbalanced force.

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 3000}{60}$$

$$= 314.16 \text{ rad/sec}$$

Natural circul frequency, $\omega_n = \sqrt{S/m}$.

$$= \sqrt{\frac{12 \times 10^5}{75}}$$

ω_n = ~~N4\cdot14~~ rad/s

$$\omega_n = 126.5 \text{ rad/s}$$

Frequency ratio, $r = \frac{\omega}{\omega_n}$.

$$= \frac{314.16}{126.5}$$

$$r = 2.48$$

eccentricity, $e = \frac{\text{stroke}}{2} = 0.04\,m$.

unbalanced force, $F_0 = m_u \omega^2 e = 2 \times 314.16^2 \times 0.04$

$$= 7895.68\ N.$$

(i) Amplitude of motion of machine:

$$x_{max} = \frac{F_0/S}{\ldots}$$

$$= \frac{7895.68/12\times10^5}{\sqrt{(1-2.48^2)^2 + (2\times0.2\times2.48)^2}}$$

$$= \frac{6.579\times10^{-3}}{5.245}$$

$$= 1.25\times10^{-3}\ m.$$

$$x_{max} = 1.25\ mm.$$

(ii) Phase angle with respect to exciting force:

$$\varphi = \tan^{-1}\left[\frac{2\xi r}{1-r^2}\right] = \tan^{-1}\left[\frac{2\times0.2\times2.48}{1-2.48^2}\right].$$

$$\varphi = -10.9° \text{ (or) } \varphi = -10.9° + 180°.$$

$$\therefore \varphi = 169°.$$

(iii) Force transmitted to the foundation: (F_t):

$$\varepsilon = \frac{F_t}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\frac{F_T}{7895.68} = \frac{\sqrt{1+(2\times0.2\times2.48)^2}}{\sqrt{(1-2.48^2)^2+(2\times0.2\times2.48)^2}}$$

$$F_T = 2120.42\ N.$$

iv) Phase angle of transmitted force with respect to exciting force: (α)

$$\alpha = \tan^{-1}(2\xi r).$$

$$\alpha = \tan^{-1}(2\times0.2\times2.48)$$

v) Phase lag of the transmitted force with respect to applied force: $(\varphi - \alpha)$.

$$\text{Phase lag, } \varphi - \alpha = \tan^{-1}\left[\frac{2\zeta r}{1-r^2}\right] - \tan^{-1}(2\zeta r).$$

$$= 169^\circ - 44.78^\circ$$

$$\varphi - \alpha = 124.22^\circ$$

S. No.	Date	Title	Page No.	Teacher's Sign

3) An industrial machine weighing 445 kg is supported on a spring with a statical deflection of 0.5 cm. If the machine has rotating imbalance of 25 kgcm, find the force transmitted at 1200 rpm and the dynamic amplitude at that speed.

Gn:

$m = 445$ kg.

$\delta = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$.

$m_u \cdot e = 25$ kg.cm.

$m_u \cdot e = 0.25$ kg.m.

$N = 1200$ rpm

Soln:

Angular velocity, $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi(1200)}{600}$

$\omega = 125.66$ rad/sec.

Natural frequency, $\omega_n = \sqrt{s/m} = \sqrt{g/\delta}$

$= \sqrt{\frac{9.81}{0.5 \times 10^{-2}}}$

$\omega_n = 44.29$ rad/s.

$\therefore$ Frequency ratio, $r = \omega/\omega_n$.

$= \frac{125.66}{44.29}$

Unbalanced exciting force, $F_0 = (m_u \cdot e)\omega^2$.

$= 0.25 \times 125.66^2$.

$F_0 = 3947.6$ N

(i) Force transmitted at 1200 rpm : (F_T):

Force transmitted when there is no damper,

no information abt 'S' given.

$$\mathcal{E} = \frac{F_T}{F_0} = \frac{1}{1 - r^2}$$

when $r > 1$, $\mathcal{E} = \frac{1}{r^2 - 1}$

$$\frac{F_T}{F_0} = \frac{1}{r^2 - 1}$$

$$\therefore \frac{F_T}{3947.6} = \frac{1}{2.837^2 - 1}$$

$\Rightarrow F_T = 560.06$ N.

(ii) Dynamic amplitute at 1200 rpm (x_{max}):

W.K.T, amplitude of vibration when there is no damper,

$$x_{max} = \frac{F_0/s}{1 - r^2}.$$

when, $r > 1$, $x_{max} = \frac{F_0/s}{r^2 - 1}$

$$= \frac{3947.6/0.5 \times 10^{-2}}{\;}$$

Finding 's' value:

$$\omega_n = \sqrt{s/m}$$

$$44.29 = \sqrt{\frac{s}{445}}$$

$$(44.29)^2 = s/445$$

$s = 8.73 \times 10^5$ N/m.

$$\therefore x_{max} = \frac{3947.6/8.73 \times 10^5}{2.837^2 - 1}.$$

$x_{max} = 6.23 \times 10^{-4}$ m.

$\therefore x_{max} = 0.623$ mm.

Result:

(i) Force transmitted at 1200 rpm, $F_T = 560.06$ N.

(ii) Dynamic amplitude at 1200 rpm, $x_{max} = 0.623$ mm

**

4) The mass of an electric motor is 120 kg and it runs at 1500 rpm. The armature mass is 35 kg and it's center of gravity lies 0.5 mm from the axis of rotation. The motor is mounted on 5 springs of negligible damping so that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the

e

$\xi = 0$.

(ii) Dynamic force transmitted to the base of the operating speed.

(iii) Natural frequency of the system.

Gn:

$m = 120$ kg.

$N = 1500$ rpm

$m_u = 35$ kg.

$e = 0.5$ mm $= 0.5 \times 10^{-3}$ m.

no. of springs $= 5$

$\zeta = 0$.

Also, $F_T = \frac{1}{11} F_0$.

$\Rightarrow \frac{F_T}{F_0} = \frac{1}{11}$

$\therefore \varepsilon = \frac{F_T}{F_0} = \frac{1}{11} = 0.091$.

Soln:

Angular velocity, $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi (1500)}{60}$

$\omega = 157.08$ rad/s.

Natural frequency, $\omega_n = \sqrt{s/m} = \sqrt{g/\delta}$.

$= 9.81$

Unbalancing exciting force, $F_0 = (m_u \cdot e)\,\omega^2$.

$= 35 \times 0.5 \times 10^{-3} \times 157.08^2$.

$= 431.79$ N.

(i) Stiffness of each spring:

In the absence of damping ($\zeta = 0$), the transmissibility is given by, $\varepsilon = \pm \frac{1}{1 - r^2}$

Assume: when, $r > 1$, $\varepsilon = \frac{1}{r^2 - 1}$.

$\varepsilon = \frac{1}{(\omega/\omega_n)^2 - 1}$.

$0.091 = \frac{1}{\left(\frac{157.08}{\omega_n}\right)^2 - 1}$

$0.091 \times \left[\frac{157.08^2}{\omega_n^2} - 1\right] = 1$.

$\frac{2245.345}{\omega_n^2} - 0.091 = 1$.

$\frac{2245.345}{\omega_n^2} = 1.091$.

$\therefore \omega_n^2 = 2058.06$

$\therefore \omega_n = 45.37$ rad/s.

W.K.T, $\omega_n = \sqrt{S/m}$

$45.37 = \sqrt{S/120}$

$45.37^2 \times 120 = S$

$S = 2.47 \times 10^5 \text{ N/m}$

$\therefore$ Combined stiffness of the springs $\}$, $S = 2.47 \times 10^5 \text{ N/m}$.

$\therefore$ Stiffness of each spring $= \frac{2.47 \times 10^5}{5}$

$= 0.494 \times 10^5 \text{ N/m}$.

(ii) Dynamic force transmitted to base (F_T):

W.K.T, $\varepsilon = \frac{F_T}{F_0}$

$F_T = 0.091 \times 431.79$

$F_T = 39.29 \text{ N}$

(iii) Natural frequency of the system : (f_n)

W.K.T, $f_n = \frac{\omega_n}{2\pi}$

$= \frac{45.366}{2\pi}$

$f_n = 7.22 \text{ Hz}$

Result :

(i) Stiffness of each spring $= 0.494 \times 10^5$ N/m

(ii) Dynamic force of transmitted to the base, $F_T = 39.29$ N

(iii) Natural frequency of the system, $f_n = 7.22$ Hz.

**

5) A single cylinder reciprocating engine of total mass 250 kg is to be installed on an elastic support which permits vibratory motion only in vertical direction. The mass of piston is 3.75 kg and it reciprocates vertically with a stroke of 150 mm. The maximum vibratory force transmitted through the elatic support to the foundation must be limitted to 500 N when the engine runs at 750 rpm and less than 500 N at all higher speeds.

(i) Find the neccessary stiffness of the elastic support and the amplitude of vibration at 800 rpm. Take frequency ratio, $r > 1$.

(ii) If the engine speed is reduced below 750 rpm, at what speed will be the transmitted force again becomes 500 N, take frequency ratio, $r < 1$.

Gn:

$m = 250\ kg.$

$m_u = 3.75\ kg.$

Stroke $= 150\ mm = 0.15\ m.$

$e = \frac{\text{Stroke}}{2} = 0.075\ m.$

$F_T = 500\ N$

$N = 750\ rpm.$

$\zeta = 0.$

Soln:

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi \times 750}{60}$

$= 78.54\ rad/s.$

Disturbing force at 750 rpm,

$F_0 = (m_u \cdot e)\,\omega^2.$

$= (3.75 \times 0.075)(78.54)^2.$

$= \cancel{1.735 \times 10^5\ N}.\ 1734.89\ N.$

(i) Stiffness of elastic support (s)

Maximum vibrating force transmitted to the foundation, $F_T = \{\text{Stiffness of elastic support}\} \times \{\text{Maximum amplitude of vibration}\}$

$F_T = S \times x_{max}.$

$\therefore F_T = S \times \frac{F_0/S}{r^2 - 1} \quad [\because r > 1\ \&\ \zeta = 0].$

$= \frac{F_0}{(\omega/\omega_n)^2 - 1}$

$x_{max} = \frac{F_0/s}{r^2-1}$

$= \frac{F_0}{s\left[\frac{\omega^2}{\omega_n^2} - 1\right]}$

$= \frac{F_0\,\omega_n^2}{s(\omega^2 - \omega_n^2)}$

$= \frac{F_0(S/m)}{S(\omega^2 - S/m)}$

$= \frac{F_0/\cancel{m}}{\frac{\omega^2 m - S}{\cancel{m}}}$

$= \frac{F_0}{\frac{\omega^2}{\omega_n^2} - 1}$

$= \frac{F_0}{\left(\frac{\omega^2 - \omega_n^2}{\omega_n^2}\right)}$

$= \frac{F_0\,\omega_n^2}{\omega^2 - \omega_n^2}$

$\omega_n = \sqrt{S/m}$

$= \frac{F_0\,(S/m)}{\omega^2 - \omega_n^2}$

$\omega_n^2 = S/m.$

$= \frac{F_0 S}{m(\omega^2 - \omega_n^2)}$

$= \frac{F_0 S}{m(\omega^2 - S/m)}$

$= \frac{F_0 S}{m\left[\frac{\omega^2 m - S}{m}\right]}$

$F_T = \frac{F_0 S}{\omega^2 m - S}.$

$F_T = S \times x \qquad x_{max} = \frac{F_0}{\ }$ — (1).

$$\Rightarrow 500 = \frac{1734.89 \times S}{250(78.54)^2 - S}$$

$$1734.89 S = 500 \left[1542132.9 - S\right].$$

$$1734.89 S = 771066450 - 500 S$$

$$2234.89 S = 771066450.$$

$$S = 345013.16 \text{ N/m}$$

$$S = 345 \times 10^3 \text{ N/m}.$$

From ①, Amplitude of vibration at 800 rpm is,

$$x_{max} = \frac{F_0}{m\omega^2 - S}$$

$$= \frac{1734.89}{250(78.54)^2 - 345 \times 10^3}$$

here, N = 800 rpm.

$$\therefore \omega = \frac{2\pi(800)}{60}$$

$$\omega = 83.79 \text{ rad/s}.$$

$$\therefore x_{max} = \frac{F_0(1734.89)}{250(83.79)^2 - (345 \times 10^3)}$$

$$x_{max} = 1.23 \times 10^{-3} \text{ m}.$$

$$x_{max} = 1.23 \text{ mm}.$$

ii) Speed at which the transmitted force again becomes 500 N:

Let, this speed be N_1 rpm (or) ω_1 rad/s,

Disturbing force, $F_0 = m_u \cdot e\,\omega_1^2$.

$$F_0 = 3.25 \times 0.075\, \omega_1^2.$$

$$F_0 = 0.244\, \omega_1^2 \text{ N}.$$

and maximum force transmitted,

$$F_T = S \times \frac{F_0}{S - m\omega_1^2} \quad [\because r < 1].$$

$$500 = (345 \times 10^3)\left[\frac{0.244(\omega_1^2)}{345 \times 10^3 - 250\omega_1^2}\right].$$

$$6 \times 10^{-3} = \frac{1}{345 \times 10^3 - 250\omega_1^2}$$

$$345 \times 10^3 - 250\omega_1^2 = \frac{\omega_1^2}{6 \times 10^{-3}}.$$

$$345 \times 10^3 - 250\omega_1^2 = 166.67\,\omega_1^2.$$

$$250\omega_1^2 = 344833.33.$$

$$\therefore \omega_1 = 37.14 \text{ rad/s}.$$

$$345 \times 10^3 = 416.67\,\omega_1^2.$$

$$\therefore \omega_1 = 28.77 \text{ rad/s}.$$

$\Rightarrow \dfrac{2\pi N_1}{60} = 28.77$ rad/s.

$\therefore N_1 = \dfrac{28.77 \times 60}{2\pi}$

$\therefore N_1 = 274.73$ rpm.

Results:

(i) Stiffness of elastic support, $s = 345 \times 10^3$ N/m.

Amplitude of vibration, $x_{max} = 1.23$ mm.

(ii) Speed at which the transmitted force again becomes 500 N, $N_1 = 274.73$ rpm.

6) Find the stiffness of each spring when a refrigerator unit having a mass of 30 kg is to be supported by three springs. The force transmitted to the supporting structure is only 10% of the impressed force. The refrigerator unit operates at 420 rpm.

Gn:

$m = 30$ kg.

no. of springs = 3.

$F_T = 10\% \, F_0$

$F_T = 0.1 F_0$

$\Rightarrow \varepsilon = \dfrac{F_T}{F_0} = 0.1$.

Soln:

Angular velocity of refrigerator unit, $\omega = \dfrac{2\pi N}{60}$

$= \dfrac{2\pi(420)}{60}$

$\omega = 43.98$ rad/s

As no damper is used, $c = 0$.

If $r < 1$, $\therefore \varepsilon = \dfrac{1}{(1-r^2)}$

$0.1 = \dfrac{1}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}$

$1 - \dfrac{\omega^2}{\omega_n^2} = \dfrac{1}{0.1}$

$1 - \dfrac{\omega^2}{\omega_n^2} = 10$

$1 - \dfrac{43.98^2}{\omega_n^2} = 10$

$1 - \dfrac{1934.24}{\omega_n^2} = 10$

$\omega_n^2(1-10) = 1934.24$.

If $r > 1$, $\varepsilon = \dfrac{1}{(r^2-1)}$

$0.1 = \dfrac{1}{\left(\omega/\omega_n\right)^2 - 1}$

$\dfrac{\omega^2}{\omega_n^2} - 1 = \dfrac{1}{0.1}$

$\dfrac{\omega^2}{\omega_n^2} - 1 = 10$.

$\dfrac{43.98^2}{\omega_n^2} - 1 = 10$.

$\dfrac{1934.24}{\omega_n^2} - 1 = 10$

$11\omega_n^2 = 1934.24$

$\therefore \omega_n = 13.26$ rad/s.

W.K.T,

$$\omega_n = \sqrt{S/m}$$

$$\Rightarrow 13.26^2 = \frac{S}{30}$$

equivalent Spring stiffness $\}$, $S = 5274.828$ N/m.

$\therefore$ stiffness on each spring $= \frac{5274.828}{3}$

$$S = 1758.276 \text{ N/m}.$$

Result:

Stiffness on each spring = 1758.276 N/m.

7) A compressor supported symmetrically on four springs has a mass of 100 kg. The mass of the reciprocating parts is 2 kg which move through a vertical stroke of 80 mm. Neglecting damping, determine the combined stiffness of the springs so that the force transmitted to the foundation is $\frac{1}{25}$ of the impressed force. The machined crankshaft rotates at 1000 rpm. When the compressor is actually supported on the springs it is found that the damping reduces the amplitude of successive free vibration by 25%. Find

(i) The force transmitted to the foundation at 1000 rpm.

(ii) The force transmitted to the foundation at resonance.

(iii) The amplitude of the vibration at resonance.

Gn:

$m = 100$ kg

$m_u = 2$ kg

stroke $= 80$ mm $= 0.08$ m $\Rightarrow e = \frac{\text{stroke}}{2} = 0.04$ m

$F_T = \frac{1}{25} F_0$.

$$\Rightarrow \varepsilon = \frac{F_T}{F_0} = 0.04$$

$N = 1000$ rpm.

Soln:

Angular velocity, $\omega = \frac{2\pi N}{60}$

$$= \frac{2\pi(1000)}{60}$$

$$\omega = 104.72 \text{ rad/s}.$$

Combined stiffness of springs :(s)

Since no damper is used, $\xi = 0$

If $r < 1$, $\varepsilon = \dfrac{1}{1 - r^2}$.

$$0.04 = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$1 - \frac{\omega^2}{\omega_n^2} = \frac{1}{0.04}$$

$$1 - \frac{\omega^2}{\omega_n^2} = 25$$

$$-\frac{\omega^2}{\omega_n^2} = 24$$

$$-(104.72)^2 = 24\,\omega_n^2$$

$$\therefore \omega_n^2 = -456.92 \text{ (Imp)}$$

If $r > 1$, $\varepsilon = \dfrac{1}{r^2 - 1}$.

$$0.04 = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\frac{\omega^2}{\omega_n^2} - 1 = \frac{1}{0.04}$$

$$\frac{\omega^2}{\omega_n^2} - 1 = 25$$

$$\frac{104.72^2}{\omega_n^2} = 26$$

$$26\,\omega_n^2 = 104.72^2$$

$$\therefore \omega_n = 20.54 \text{ rad/s}$$

$$\therefore \omega_n = 20.54 \text{ rad/s}$$

$$\Rightarrow \sqrt{S/m} = 20.54$$

$$S/100 = 20.54^2$$

$$\therefore S = 42177.99 \text{ N/m}$$

$\therefore$ Combined stiffness of the springs, $\Big\}\; S = 42177.99$ N/m.

(ii) Force transmitted to the foundation at 1000 rpm (F_T)

Frequency ratio, $r = \dfrac{\omega}{\omega_n}$

$$= \frac{104.72}{20.54}$$

$$r = 5.1$$

Since the damping reduces the amplitude of successive free vibration by 25%.

$\therefore$ Final amplitude of vibration, $x_1 = 0.75\,x_0$.

$$\Rightarrow \ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\ln\left(\frac{x_0}{0.75\,x_0}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.2876 = \frac{6.28\,\xi}{\sqrt{1-\xi^2}}$$

$$0.2876\sqrt{1-\xi^2} = 6.28\,\xi$$

$$0.046^2(1-\xi^2) = \xi^2$$

$$2.1\times10^{-3} - 2.1\times10^{-3}\,\xi^2 = \xi^2$$

$$2.1\times10^{-3} = 1.0021\,\xi^2$$

$$\therefore \xi = 0.046$$

W.K.T, Transmissibility when damping is present,

$$\varepsilon = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{\sqrt{1+[2(0.046)(5.1)]^2}}{\sqrt{(1-5.1^2)^2+[2(0.046)(5.1)]^2}}$$

$$= \frac{1.105}{25.01}$$

$\varepsilon = 0.0442$.

W.K.T, $e = 0.04\ m$

$\therefore F_0 = m_u \omega^2 e$.

$= 2 \times 104.72^2 \times 0.04$.

$F_0 = 877.3\ N$

Force transmitted to the foundation,

W.K.T, $\varepsilon = \frac{F_T}{F_0}$

$0.0442 = \frac{F_T}{877.3}$

$F_T = 38.78\ N$

ii) Force transmitted to the foundation at resonance:

At resonance, $r = \frac{\omega}{\omega_n} = 1$.

$$\therefore \varepsilon = \frac{\sqrt{1+(2\zeta)^2}}{2\zeta}$$

$$= \frac{\sqrt{1+(2(0.046))^2}}{2(0.046)}.$$

$\varepsilon = 10.92$

Maximum unbalanced force on the machine due to reciprocating parts at resonance.

$\therefore F_0 = m_u \omega_n^2 \cdot e$. $(\omega = \omega_n)$

$= 2 \times 20.54^2 \times 0.04$

$F_0 = 33.75\ N$.

Force transmitted to foundation at resonance,

W.K.T, $\varepsilon = \frac{F_T}{F_0}$

$10.92 = \frac{F_T}{33.75}$

$\therefore F_T = 368.56\ N$.

iii) Amplitude of vibration at resonance:

W.K.T,

Amplitude of forced vibration at

$$\text{resonce} = \frac{\text{Force transmitted at resonance}}{\text{Combined stiffness}}$$

$$= \frac{368.56}{42177.99}$$

$$= 8.73 \times 10^{-3}\ m$$

$$= 8.73\ mm.$$

Result:

(i) Force transmitted to the foundation at 1000 rpm

$F_T = 38.78$

(ii) Force trasnitted to the foundation at resonance, $F_T = 368.56$ N.

(iii) Amplitude of forced vibration at resonance = 8.73 mm

Combined stiffness of the springs, S = 42177.99 N/m

0/08/2016 Tuesday

5. MECHANISM for CONTROL

Part - I / Governors:

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load eg. when the load on the engine increases, it's speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

A little consideration will show, that when the load increases, the configuration of governor changes and a valve is moved to increase the supply of the working fluid conversely, when the

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Result:

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$F_T = 38.78$

(ii) Force trasnitted to the foundation at

resonance, $F_T = 368.56\ N$.

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Combined stiffness of the springs, S = 42177.99 N/m

?/08/2016 Tuesday

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A little consideration will show, that when the load increases, the configuration of governor changes and a valve is moved to increase the supply of the working fluid conversely, when the load decreases, engine speed increases and the governor decreases the supply of working fluid.

Types of Governors:

1) Centrifugal governors.
2) Inertia governors.

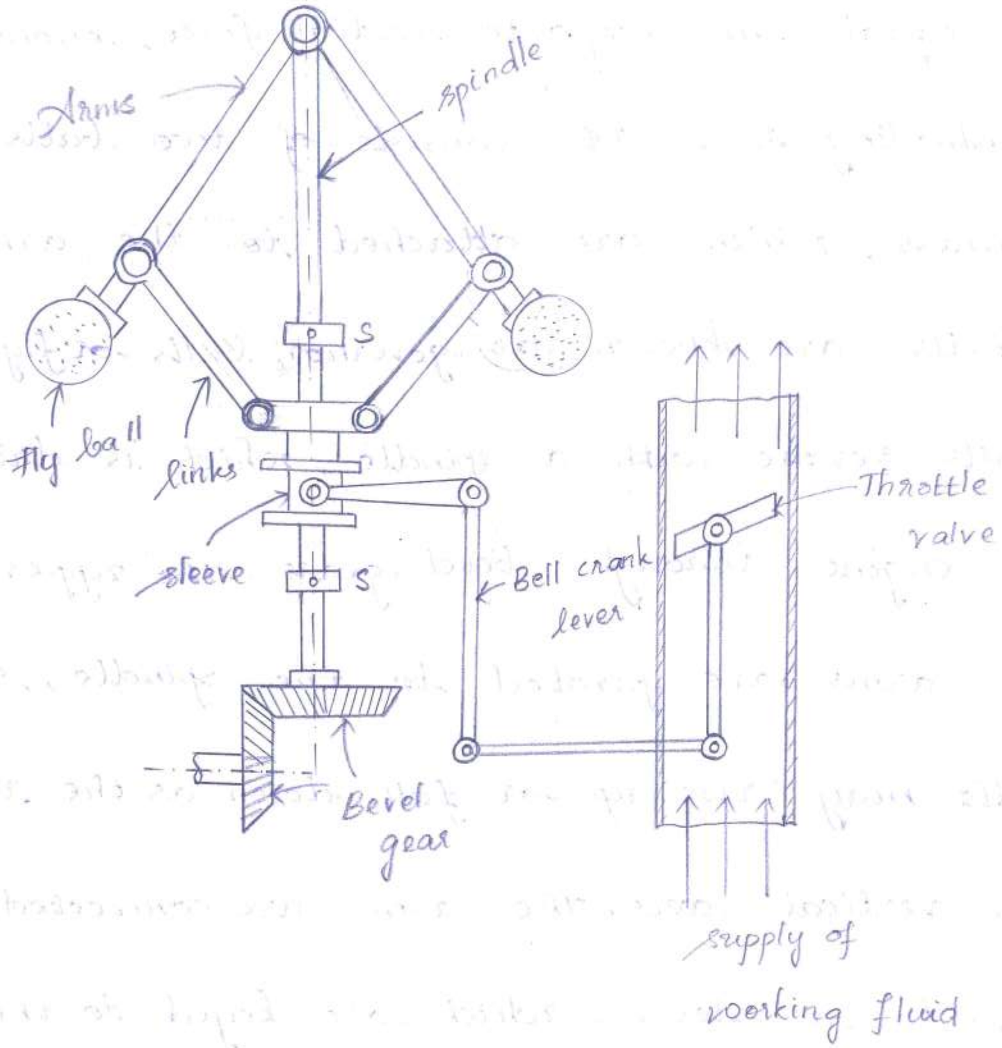
*2m

Classification of Centrifugal governors:

- Centrifugal governor.
  - Pendulum type.
    - Watt governor
  - loaded type.
    - Dead weight governors
      - Porter governor
      - Proell governor
    - Spring controlled governor
      - Hartnell governor
      - Hartung governor
      - Wilson-Hartnell governor
      - Pickering governor.

Centrifugal governors:

The centrifugal governors are based on balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force. It consists of two balls of equal mass, which are attached to the arms. These balls are known as governor balls or fly balls. The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which are keyed to the spindle. This sleeve revolves with the spindle but can slide up and down. The balls and sleeves rises when the spindle speed increases and falls when the speed decreases. In order to limit the travel of sleeve, two stops are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the fuel decreases as speed increases and vice versa. Fuel supply increases with increased load and vice versa.



Watt Governor:

The simplest form of a centrifugal governor is a Watt governor. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following 3 ways:

1) The pivot 'P', may be on the spindle axis as shown in fig(a)

2) The pivot 'P', may be offset from the spindle axis and the arms when produced intersect at 'O' as shown in fig(b).

3) The pivot 'P', may be offset, but the arms cross the axis at 'O' as shown in fig (c).

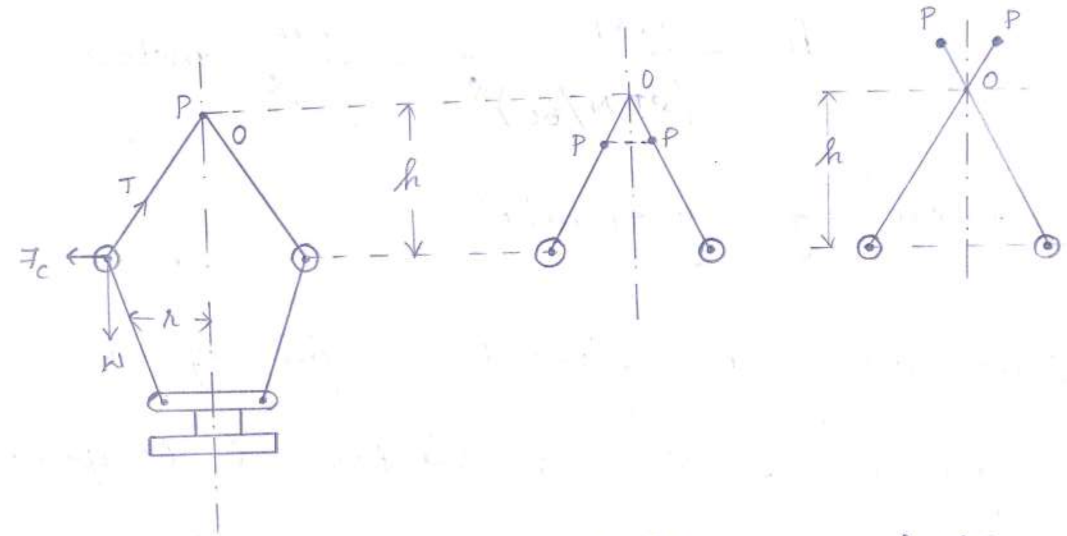


fig (a) fig (b) fig (c)

Let, $m \rightarrow$ mass of the ball in kg

$W \rightarrow$ Weight of the ball in $N = m \cdot g$.

$T \rightarrow$ Tension in the arms in N.

$\omega \rightarrow$ Angular velocity of the arm and ball about the spindle axis in rad/s;

$r \rightarrow$ radius of the path of rotation of the ball (ie) horizontal distance from the center of the ball to the spindle axis in 'm'.

$F_c \rightarrow$ Centrifugal force acting on the ball in 'N'

$$F_c = m\omega^2 r.$$

$h \rightarrow$ height of the governor in 'm'.

$$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ meters}$$

where, $g = 9.81\ m/s^2$.

Pbm:

1) Calculate the vertical height of a Watt governor, when it rotates at 60 rpm. Also find the change in vertical height when it speed increases to 61 rpm.

Gn:

$N_1 = 60$ rpm

$N_2 = 61$ rpm.

Soln:

(i) Initial height:

$$h_1 = \frac{895}{N_1^2}$$

$$= \frac{895}{60^2}$$

$h_1 = 0.2486$ m.

(ii) Final height:

$$h_2 = \frac{895}{N_2^2}$$

$$= \frac{895}{61^2}$$

$h_2 = 0.2405$ m.

$\therefore$ change in vertical height $= h_1 - h_2$

$= 0.2486 - 0.2405$

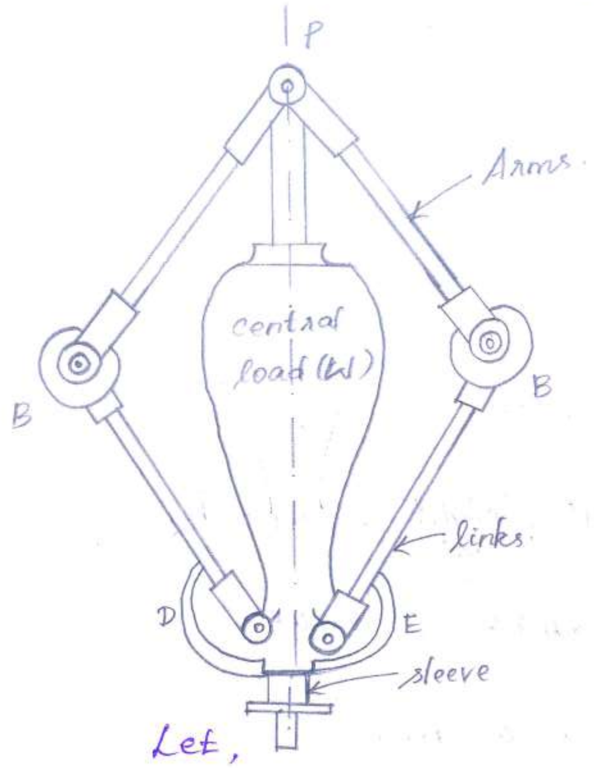
$= 8 \times 10^{-3}$ m.

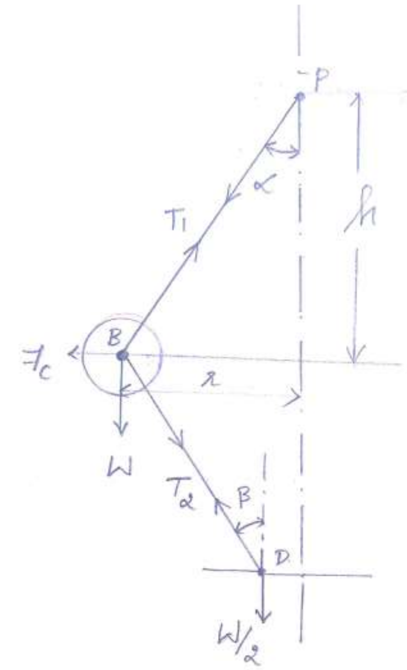
$= 8$ mm.

Porter Governor:

Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in figure. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise at any predetermined level.

Porter Governor :





Let,

$m \rightarrow$ mass of each ball in kg.

$W \rightarrow$ Weight of each ball in N.

$M \rightarrow$ Mass of the central load in kg

$W \rightarrow$ Weight of the central load in N

$$W = m.g.$$

$r \rightarrow$ radius of rotation in 'm'.

$h \rightarrow$ height of the governor in 'm'.

$N \rightarrow$ Speed of the balls in 'rpm'

$\omega \rightarrow$ angular speed of the ball in 'rad/s'

$$\omega = \frac{2\pi N}{60} \text{ rad/s}.$$

$F_c \rightarrow$ centrifugal force acting on the ball in 'N'.

$T_1 \rightarrow$ force in the arm in 'N'.

$T_2 \rightarrow$ force in the link in 'N'.

$\alpha \rightarrow$ Angle of inclination of the arm (upper link) to the vertical.

$\beta \rightarrow$ Angle of inclination of the link (lower link) to the vertical.

NOTES :

1) When the length of arms are equal to the length of links and if the points 'P' and 'D' lie on the same vertical line,

then, $\tan\alpha = \tan\beta$

(or)

$$q = \frac{\tan\beta}{\tan\beta\alpha} = 1.$$

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}.$$

2) When the load at sleeve moves up and down in the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.

I_1. $F \rightarrow$ frictional force acting on the (Me M) then,

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}.$$

$$N^2 = \frac{mg + (M.g \pm F)}{m.g} \times \frac{895}{h} \quad ; (\text{when } q = 1)$$

<u>Pbms:</u>

1) The porter governor has equal arms each 250mm long and pivoted on the axis of rotation. Each ball has a mass of 5kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150mm when the governor begins to lift and 200mm when the governor is at maximum speed. Find the minimum and maximum speed and range of speed of the governor.

Soln:

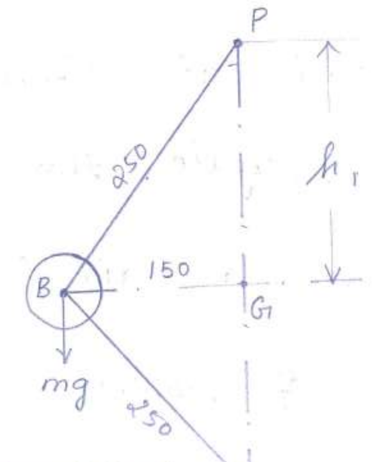


fig (a): Minimum speed

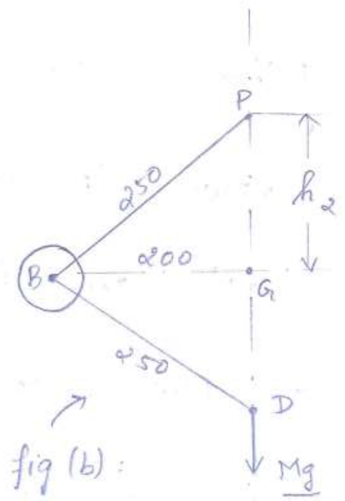


fig (b):

Gn:

From dig,

$BP = BD = 250mm = 0.25m.$

$m = 5\ kg.$

$M = 25\ kg.$

$r_1 = 150\ mm = 0.15m.$

$r_2 = 200mm = 0.2m.$

Soln:

Minimum speed when $r_1 = BG = 0.15m$:

Let, $N_1 \rightarrow$ Minimum speed.

From fig (a), we find that height of the governor,

~~h~~ From ΔBPG,

$$PB^2 = BG^2 + PG^2 \Rightarrow PG^2 = PB^2 - BG^2.$$

$$\therefore h_1^2 = 250^2 - 150^2 = 40000\ mm.$$

$$\Rightarrow h_1 = 0.2\ m$$

When, the lenght of arms = length of links and points 'P' and 'D' lie on same vertical line,

$$\therefore N_1^2 = \frac{m+M}{m} \times \frac{895}{h}.$$

$$= \frac{5+25}{5} \times \frac{895}{0.2}$$

$$N_1^2 = 26850 \implies N_1 = 163.85 \text{ rpm}$$

(ii) Maximum speed when $r_2 = BG = 0.2$ m.

Let, $N_2 \longrightarrow$ Maximum speed

From fig (b),

From Δ BGP,

$$BP^2 = PG^2 + BG^2$$

$$250^2 = h_2^2 + 200^2$$

$$\therefore h_2 = 150 \text{ mm}$$

$$\implies h_2 = 0.15 \text{ m}$$

$$\therefore N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2}$$

$$= \frac{5+25}{5} \times \frac{895}{0.15}$$

$$N_2^2 = 35800.$$

$$\therefore N_2 = 189.2 \text{ rpm}$$

Range of speed:

W.K.T, Range of speed $= N_2 - N_1$.

$$= 189.2 - 163.85$$

$$= 25.35 \text{ rpm}.$$

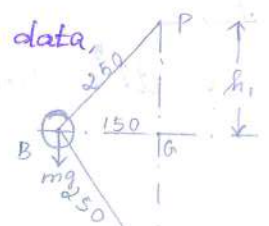
NOTE:

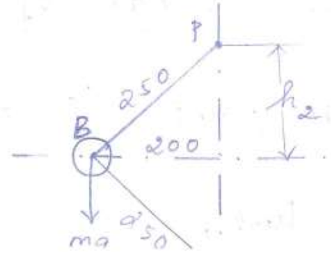
3) When the points 'P' and 'D' are not lying on the same vertical line, then

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

) The arms of a porter governor are each 250mm long and pivoted on the governor axis. The mass of each ball is 5kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150mm when the scale begins to rise and reaches a value of 200mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, Determine how the speed range is modified.

Sol: From data,





From dig,

$BP = BD = 250\,mm = 0.25\,m$

$m = 5\,kg$

$M = 30\,kg$.

$r_1 = 150\,mm = 0.15\,m$

$r_2 = 200\,mm = 0.2\,m$.

Soln:

Minimum speed when $r_1 = BG = 0.15\,m$:

Let,

$N_1 \longrightarrow$ Minimum speed.

From fig (a),

In ΔBGP,

$PB^2 = BG^2 + GP^2$.

$250^2 = 150^2 + h_1^2$.

$\therefore h_1^2 = 40000\,mm$.

$\therefore h_1 = 200\,mm$.

$h_1 = 0.2\,m$

When length of arms = length of links and when points 'P' and 'D' lie in the same vertical line,

$$N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1}.$$

$$= \frac{5+30}{5} \times \frac{895}{0.2}.$$

$N_1^2 = 31325$

$\therefore N_1 = 177$ rpm.

Maximum speed when $r_2 = BG = 0.2\,m$:

Let, $N_2 \longrightarrow$ maximum speed.

From fig. (b),

From ΔBGP,

$BP^2 = BG^2 + GP^2$.

$\therefore h_2^2 = 250^2 - 200^2$.

$\Rightarrow h_2 = 0.15\,m$.

$$\therefore N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2}.$$

$$N_2^2 = \frac{5+30}{5} \times \frac{895}{0.15}$$

$\Rightarrow N_2 = 204.4$ rpm.

$\therefore$ Range of speed $= N_2 - N_1$

$= 204.4 - 177$

$= 27.4$ rpm.

Speed range when friction at sleeve is of 20 N of load (When $F = 20$ N)

W.K.T, when the sleeve moves downwards, the frictional force 'F' acts upwards and the minimum speed is given by,

$$N_1^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_1}$$

$$= \frac{5\times9.81 + ((30\times9.81) - 20)}{5\times9.81} \times \frac{895}{\cancel{h_1}\ 0.2}$$

$\therefore N_1 = 172$ rpm.

We know that, when the sleeve move upwards, the frictional force 'F' acts downwards and the maximum speed is given by,

$$N_2^2 = \frac{mg + (Mg + F)}{mg} \times \frac{895}{h_2}$$

$$N_2^2 = \frac{5\times9.81 + ((30\times9.81) + 20)}{5\times9.81} \times \frac{895}{0.15}$$

$\therefore N_2 = 210$ rpm.

$\therefore$ Speed range of governor $= N_2 - N_1$.

$= 210 - 177$

$= 38$ rpm.

3) An engine governor of porter type the upper and lower arms are 200 mm and 250 mm resp. and pivoted on the same line. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 24 N at the sleeve. If the limitting inclinations of the upper arms to the vertical are 30° and 40°, find taking friction into account, the range of speed of governor.

Gn: (From data):

fig: Minimum position

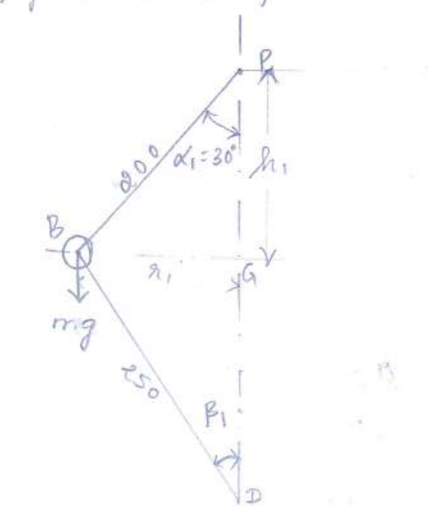
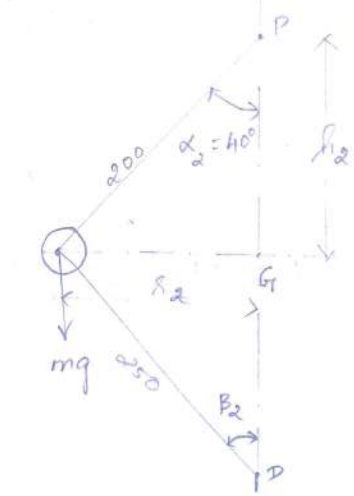


fig (b) Maximum position.



From fig,

$BP = 200\,mm = 0.2\,m$.

$BD = 250\,mm = 0.25\,m$.

$M = 15\,kg$.

$m = 2\,kg$

$F = 24\,N$.

$\alpha_1 = 30°$, $\alpha_2 = 40°$. (upper arm)

First of all, let us find the minimum and maximum speed of the governor.

The minimum and maximum position of the governor are shown in fig (a) & fig (b).

Let,

$N_1 \rightarrow$ Minimum speed

$N_2 \rightarrow$ Maximum speed.

From fig (a)

$$\sin 30° = \frac{BG}{BP} = \frac{r_1}{BP}.$$

$$\therefore \sin 30° = \frac{r_1}{0.2}$$

$$\therefore r_1 = 0.1\,m.$$

Height of governor:

$$\cos 30° = \frac{PG}{BP}$$

D

$$\therefore h_1 = \cos 30° \times 0.2.$$

$$h_1 = 0.1732\,m.$$

By using pythogoras thm,

from fig. (a), from ΔBGD,

$$BD^2 = DG^2 + BG^2.$$

$$\therefore DG = \sqrt{BD^2 - BG^2}$$

$$= \sqrt{(0.25)^2 - (0.1)^2}$$

$$DG = 0.23\,m.$$

From fig. (a),

$$\tan \beta_1 = \frac{BG}{DG}$$

$$= \frac{0.1}{0.23}$$

$$\tan \beta_1 = 0.435$$

|||ly, $\tan \alpha_1 = \tan 30° = 0.5774$.

$$\therefore q_1 = \frac{\tan \beta_1}{\tan \alpha_1}$$

$$= \frac{0.435}{0.5744}.$$

$$q_1 = 0.757$$

We know that, when sleeve mores downwards, the frictional force acts upwards and

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - 7}{2}\right)(1+q_1)}{m.g} \times \frac{895}{h_1}.$$

$$= \frac{2\times 9.81 + \left(\frac{15\times 9.81 - 24}{2}\right)(1+0.753)}{2\times 9.81} \times \frac{895}{0.1732}.$$

$\therefore N_1 = 183.3$ rpm.

From fig. (b),

$$\sin 40^\circ = \frac{BG}{BP} = \frac{r_2}{0.2}.$$

$\therefore r_2 = 0.1285$ m.

Height of governor: (h_2).

$$\cos 40^\circ = \frac{PG}{BP}.$$

$$\cos 40^\circ = \frac{h_2}{0.2}$$

$$h_2 = 0.1532 \text{ m}.$$

By Pythogoras thm,

from fig. (a), from ΔBGD

$$BD^2 = DG^2 + BG^2.$$

$$DG = \sqrt{BD^2 - BG^2}$$

$$= \sqrt{0.25^2 - 0.1285^2}.$$

$$DG = 0.214 \text{ m}.$$

From, fig (b)

$$\tan \beta_2 = \frac{BG}{DG}.$$

$$= \frac{0.1285}{0.214}.$$

$$\tan \beta_2 = 0.6$$

$$\tan \alpha_2 = \tan 40^\circ = 0.8391.$$

$$\therefore q_2 = \frac{\tan \beta_2}{\tan \alpha_2}$$

$$= \frac{0.6}{0.8391}.$$

$$q_2 = 0.715$$

Now, when sleeve move up, '7' acts downwards and so

Max. speed, $$(N_2)^2 = \frac{m.g + \left(\frac{Mg + 7}{2}\right)(1+q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2\times 9.81 + \left(\frac{15\times 9.81 + 24}{2}\right)(1+0.715)}{2\times 9.81} \times \frac{895}{0.1532}$$

$N_2 = 222.57$ rpm.

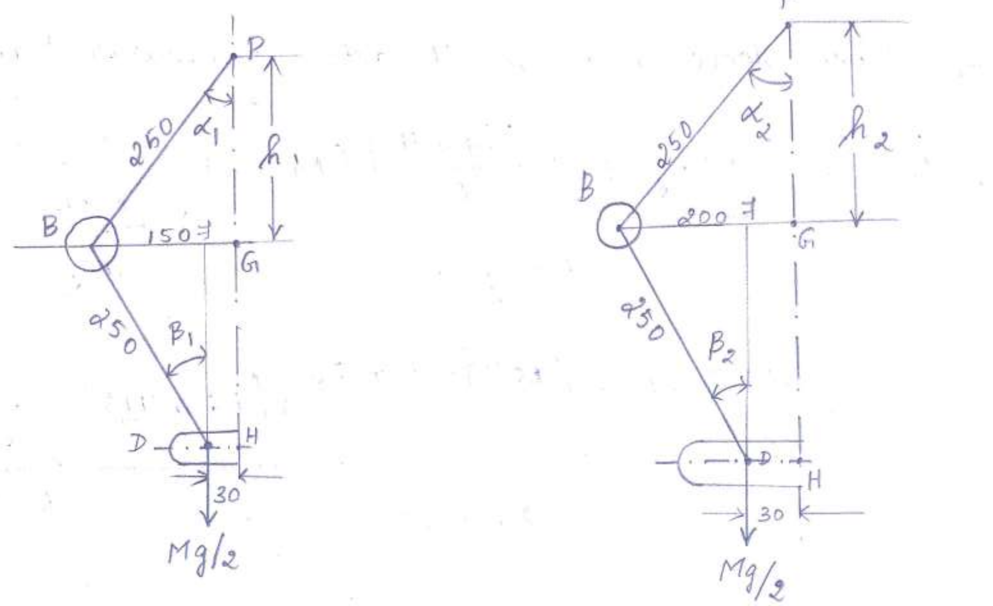
$\therefore$ Range of speed $= N_2 - N_1$

$= 222.57 - 183.3.$

Range of speed $= 39.27$ rpm.

5) A porter governer has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of governor.

Gn :



(a) Minimum position (b) Maximum position.

From fig,

BG = 150 (fig a) ; BG = 200 (fig b).

Let, $N_1 \rightarrow$ Minimum speed when $r_1 = BG = 150mm$

$N_2 \rightarrow$ Maximum speed when $r_2 = BG = 200 mm$

From fig. (a),

Height of governor (h_1):

from Δ BGP,

By pythogoras thm, $BP^2 = PG^2 + GB^2$.

$$(0.25)^2 = h_1^2 + 0.15^2.$$

$$\therefore h_1 = 0.2 m.$$

Also, $BF = BG - FG$

$BF = 0.150 - 0.03$

$\therefore BF = 0.12 m.$

From Δ BFD,

By pythogoras thm, $BD^2 = BF^2 + FD^2$.

$$0.25^2 = 0.12^2 + FD^2.$$

$$\therefore FD = 0.2193 m.$$

Also, $\tan \alpha_1 = \frac{BG}{PG} = \frac{0.15}{0.2} = 0.75$

$$\tan \beta_1 = \frac{BF}{FD} = \frac{0.12}{0.2193} = 0.548.$$

$$\therefore q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

W.K.T,

$$N_1^2 = \frac{m + \frac{M}{2}(1+q_1)}{m} \times \frac{895}{h_1}$$ (case is applicable to note (3))

$$= \frac{5 + \frac{50}{2}(1+0.731)}{5} \times \frac{895}{0.2}$$

$\therefore N_1 = 208$ rpm.

From fig (b),

Height of governor (h_2):

Take ΔBGP and apply pythogoras thm,

$BP^2 = PG^2 + GB^2$.

$0.25^2 = h_2^2 + 0.2^2$.

$\therefore h_2 = 0.15$ m.

Also, $BF = BG - FG$.

$BF = 0.2 - 0.03$

$\therefore BF = 0.17$ m.

From ΔBFD,

By pythogoras thm, $BD^2 = BF^2 + FD^2$.

$0.25^2 = \cancel{0.17}^2 + FD^2 + 0.17^2$.

$\therefore FD = 0.183$m.

Also, $\tan \alpha_2 = \frac{BG}{PG} = \frac{0.2}{0.15} = 1.333$

$\tan \beta_2 = \frac{BF}{FD} = \frac{0.17}{0.183} = 0.93$.

$\therefore q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$.

W.K.T,

$$N_2^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2}$$

$$= \frac{5 + \frac{50}{2}(1+0.7)}{5} \times \frac{895}{0.15}$$

$\therefore N_2 = 238$ rpm.

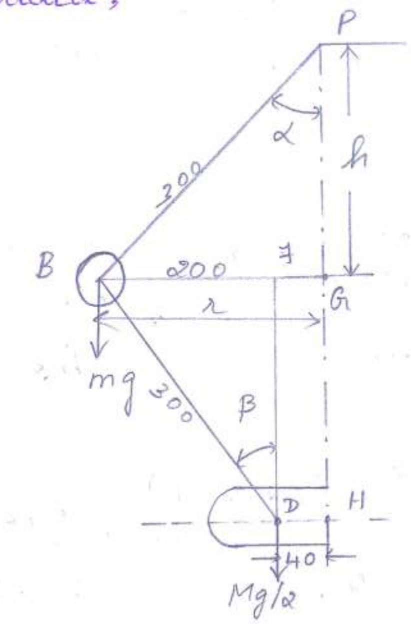
$\therefore$ Range of speed $= N_2 - N_1 = 238 - 308$.

$\therefore$ Speed range $= 30$ rpm

6) The arms of a porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200mm.

If the friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position?

Gn: From data,



Gn:

$m = 10$ Kg.

$M = 70$ Kg.

$BP = BD = 300mm = 0.3 m$.

$DH = 40mm = 0.04m$.

$r = BG = 200mm = 0.2m$

Soln:

Equillibrium speed when the radius of rotation

$r = BG = 0.2m$:

Let, $N \longrightarrow$ equillibrium speed.

height of the governor (h):

By pythogoras thm in Δ BPG,

$BP^2 = BG^2 + GP^2$.

$0.3^2 = 0.2^2 + h^2$.

$\therefore h = 0.224 m$.

$BF = BG - FG$

$= 0.2 - 0.04$.

$BF = 0.16 m$

From Δ BFD,

By pythogoras thm,

$BD^2 = BF^2 + FD^2$.

$0.3^2 = 0.16^2 + FD^2$.

$\therefore FD = 0.254$ m.

$$\tan\alpha = \frac{BG}{PG} = \frac{0.2}{0.224} = 0.893$$

$$\tan\beta = \frac{BF}{FD} = \frac{0.16}{0.254} = 0.63.$$

$$\therefore q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.893} = 0.705$$

Now, equillibrium speed is given by,

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{70}{2}(1+0.705)}{10} \times \frac{895}{0.224}$$

Range of speed when friction is equivalent to load of 20 N at the sleeve : (ie, when, $F = 20N$)

Let,

$N_1 \rightarrow$ Minimum equillibrium speed.

$N_2 \rightarrow$ Maximum equillibrium speed.

W.K.T, when sleeve moves ~~upwards~~ down, the frictional force (F) acts ~~downwards~~ up & the minimum equillibrium speed is,

$$N_1^2 = \frac{m \cdot g + \left(\frac{M \cdot g - F}{2}\right)(1+q)}{m \cdot g} \times \frac{895}{h}$$

$$= \frac{(10 \times 9.81) + \left(\frac{70 \times 9.81 - 20}{2}\right)(1 + 0.705)}{10 \times 9.81} \times \frac{895}{0.224}$$

$\therefore N_1 = 165$ rpm.

Also when sleeve moves up, 'F' acts downward

$\therefore$ Max. equillibrium speed is,

$$N_2^2 = \frac{m \cdot g + \left(\frac{Mg + F}{2}\right)(1+q)}{m \cdot g} \times \frac{895}{h}.$$

$$= \frac{(10 \times 9.81) + \left(\frac{70 \times 9.81 + 20}{2}\right)(1 + 0.705)}{10 \times 9.81} \times \frac{895}{0.224}$$

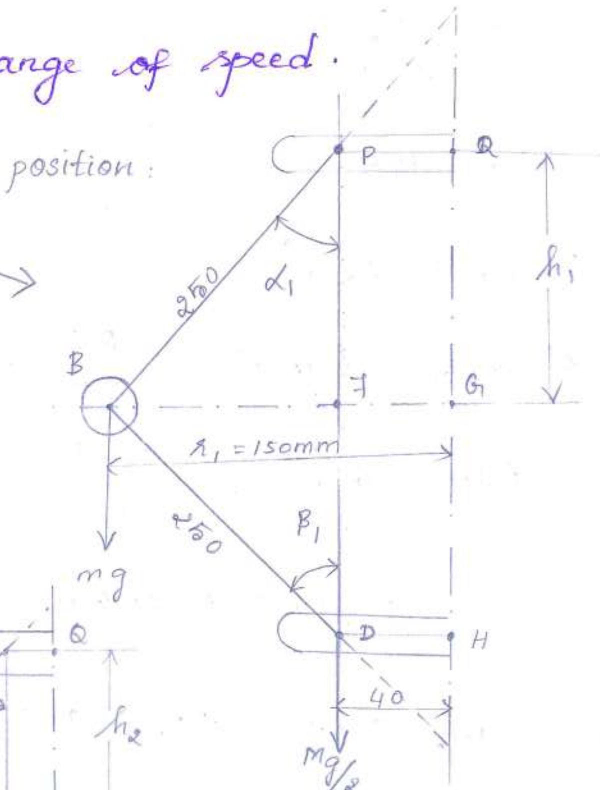
Range of speed $= N_2 - N_1$.

$= 169 - 165$

$= 4$ rpm.

7) A loaded porter governor has 4 links each 250mm long, two revolving masses each of 3 kg and a central dead weight of mass 20 kg. All the links are attached to respective sleeves at radial distances of 40 mm from the axis of rotation. The masses revolve at a radius of 150mm at minimum speed and a radius of 200 mm at maximum speed. Determine the range of speed.

fig. (a) Minimum position.



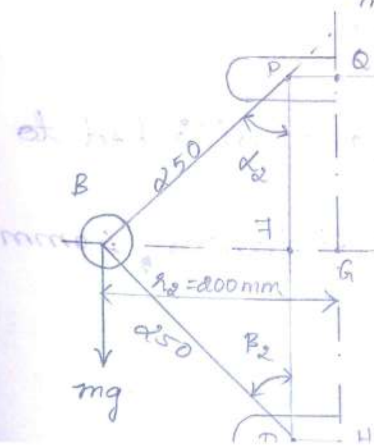


fig (b) Maximum position.

Gn:

$BP = BD = 250$ mm.

$m = 3$ kg.

$M = 20$ kg

$PQ = DH = 40$ mm.

$r_1 = 150$ mm

$r_2 = 200$ mm.

Soln:

From fig (a), $BF = BG - GF$

$= 150 - 40$

$BF = 110$ mm.

$$\sin\alpha_1 = \frac{BF}{BP}$$

$$= \frac{110}{250}$$

$\sin\alpha_1 = 0.44$

$\therefore \alpha_1 = 26.1°$.

from Δ^{le} OBG, $\tan\alpha_1 = \frac{BG}{OG} = \frac{BG}{h_1}$.

$$\tan 26.1° = \frac{0.15}{h_1}$$

$\therefore h_1 = 0.306$ m.

Since all the links are attached to respective sleeves at equal distances 40mm from the axis of rotation.

$\therefore \tan\alpha_1 = \tan\beta_1$.

(or)

$q = 1$.

W.K.T,

$$N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1}.$$

$$= \frac{3+20}{3} \times \frac{895}{0.306}.$$

$N_1 = 150$ rpm.

From fig (b),

$BF = BG - GF$

$= 200 - 40$

$= 160$ mm

$BF = 0.16$ m

$$\sin\alpha_2 = \frac{BF}{BP}$$

$$\sin\alpha_2 = \frac{0.16}{0.25}$$

$\therefore \sin\alpha_2 = 0.64$

$\therefore \alpha_2 = 39.79°$.

from Δ^{le}, OBG, $\tan\alpha_2 = \frac{BG}{OG} = \frac{BG}{h_2}$

$$\Rightarrow \tan 39.79° = \frac{0.2}{h_2}.$$

$h_2 = 0.24$ m.

Since all the links are attached to the respective ~~same~~ sleeves at equal distances 40mm from the axis of rotation.

$\therefore \tan \alpha_2 = \tan \beta_2$.

$\therefore q = 1$.

$$\therefore N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2}.$$

$$= \frac{3+20}{3} * \frac{895}{0.24}.$$

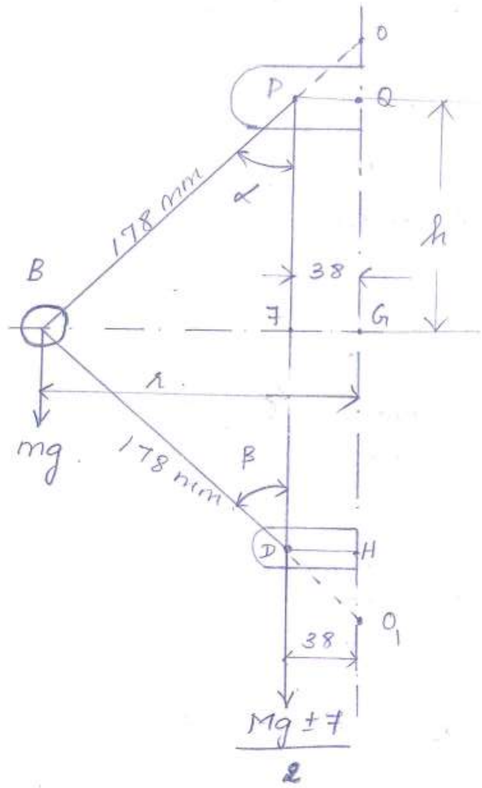
$N_2 = 169.~~13~~$ rpm.

$\therefore$ Range of speed $= N_2 - N_1$

$= 169 - 150$

$= 19$ r.p.m.

8) All the arms of a porter governor are 178mm long and are hinged at a distance of 38mm from the axis of rotation. The mass of each ball is 1.15 kg and mass of the sleeve is 20 kg. The governor sleeve begins to raise at 280 rpm when the links are at an angle of 30° to the vertical. Assuming the friction force to be constant, determ the minimum and maximum speed of rotation when the inclination of the arms to the vertical is 45°.



Gn :

BP = BD = 178 mm.

PQ = DH = 38 mm.

m = 1.15 kg

M = 20 kg

N = 280 ~~kg~~ rpm

$\alpha = \beta = 30°$.

Soln:

(i) Friction force :

$\therefore$ from $\Delta PB7$, $\sin\alpha = \dfrac{B7}{BP}$.

$B7 = \sin\alpha \cdot (178)$.

$\therefore B7 = 89$ mm.

$r = BG = B7 + 7G$

$r = 89 + 38$

$\therefore r = 127$ mm.

from ΔOBG,

$\tan\alpha = \dfrac{BG}{OG} = \dfrac{0.127}{h}$.

$\therefore \tan 30^\circ = \dfrac{0.127}{h}$.

$\therefore h = 0.22$ m.

$\tan\alpha = \tan\beta \Rightarrow q = 1$

$\therefore$ W.K.T,

$$N^2 = \frac{mg + (Mg \pm 7)}{m \cdot g} \times \frac{895}{h}.$$

$$280^2 = \frac{(1.15 \times 9.81) + ((20 \times 9.81) \pm 7)}{1.15 \times 9.81} \times \frac{895}{0.22}.$$

$217.41 = 10.96 + 196.2 \pm 7$.

$\pm 7 = 10.25$ N.

(ii) Minimum and Maximum speed when $\alpha = \beta = 45^\circ$.

here, $\alpha = 45^\circ$.

$\therefore \sin 45^\circ = \dfrac{B7}{BP}$.

$\therefore B7 = BP \cdot \sin 45^\circ = 178 \sin 45^\circ$.

$\therefore B7 = 125.86$ mm.

$r = B7 + 7G$

$= 125.86 + 38$

$\therefore r = 164$ mm.

from ΔOBG, $\tan 45^\circ = \dfrac{BG}{OG} = \dfrac{BG}{h}$.

$\therefore h = \dfrac{0.164}{\tan 45^\circ}$

$h = 0.164$ m.

Let,

$N_1 \rightarrow$ Minimum speed.

$N_2 \rightarrow$ Maximum speed.

W.K.T, $N_1^2 = \dfrac{mg + (Mg - 7)}{mg} \times \dfrac{895}{h}$.

$$= \frac{(1.15 \times 9.81) + ((20 \times 9.81) - 10.25)}{1.15 \times 9.81} \times \frac{895}{0.164}$$

$= 309$ rpm

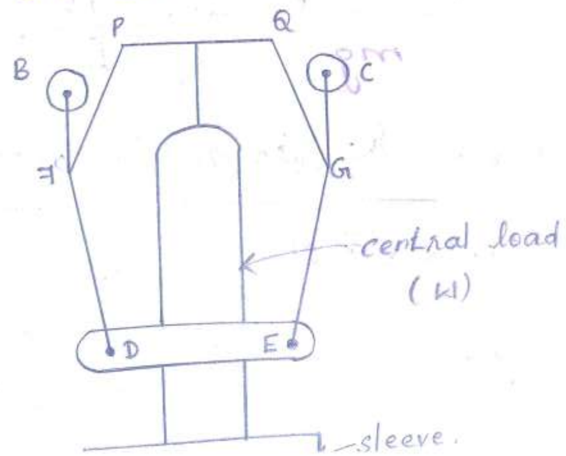
lly, $N_2^2 = \frac{mg+(Mg+F)}{mg} \times \frac{895}{h}$.

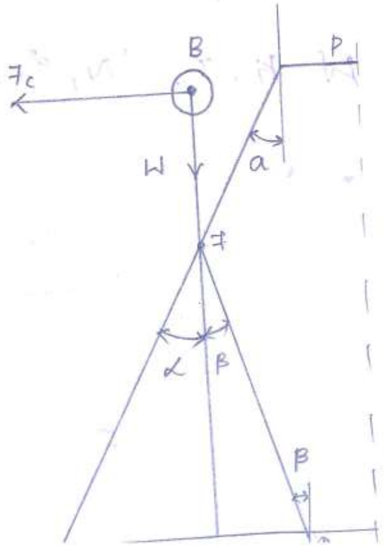
$$= \frac{(1.15\times 9.81)+((20\times 9.81)+10)}{1.15\times 9.81}\times\frac{895}{0.164}$$

∴ $N_2 = 324$ r.p.m

Proell Governor:

The proell governer has the balls fixed at B and C to the extension of the links DF and EG as shown in fig (a). The arms FP and GQ are pivoted at P and Q respectively. Consider the equillibrium of the forces on one-half of the governor as shown in fig(b). The instantaneous centre (I) lies on the intersection of the line PF produced and the line D drawn $\perp^r$ to the spindle axis. The $\perp^r$ BM is drawn on ID.





Taking moments about I, using the same rotation as discussed in art (Porter governer).

$$\boxed{F_C\times BM = \left[(W\times IM)+\left(\frac{W}{2}\, ID\right)\right] = \left[(mg\times IM)+\left(\frac{Mg}{2}\, ID\right)\right]} \quad \text{— ①}$$

$$F_c = \left(m.g\times\frac{IM}{BM}\right)+\left(\frac{M.g}{2}\times\frac{(IM+MD)}{BM}\right) \quad [\because ID = IM+M$$

Multiplying and dividing by FM, We have.

$$\frac{FM}{FM}\times F_c = \frac{FM}{BM}\left[mg\times\frac{IM}{FM}+\frac{M.g}{2}\left(\frac{IM+MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[mg\times\tan\alpha+\frac{M.g}{2}(\tan\alpha+\tan\beta)\right].$$

$$F_c = \frac{FM}{BM}\tan\alpha\left[mg+\frac{Mg}{2}\left(1+\frac{\tan\beta}{\tan\alpha}\right)\right].$$

We know that,

$$F_c = m\omega^2 r\ ;\ \tan\alpha = \frac{r}{h}\ ;\ q = \frac{\tan\beta}{\tan\alpha}.$$

$$\therefore\ m\omega^2 r = \frac{FM}{BM}\times\frac{r}{h}\left[m.g+\frac{Mg}{2}(1+q)\right].$$

$$\Rightarrow \boxed{\omega^2 = \frac{FM}{BM}\left[\frac{m+M/2\,(1+q)}{m}\right] g/h} \quad \text{— ②}$$

substituting, $\omega = \frac{2\pi N}{60}$; $g = 9.81\ m/s^2$,

we get, $\boxed{N^2 = \frac{FM}{BM}\left[\frac{m+M/2\,(1+q)}{m}\right]\frac{895}{h}}$ — ③

lower & upper a
are of diff. l

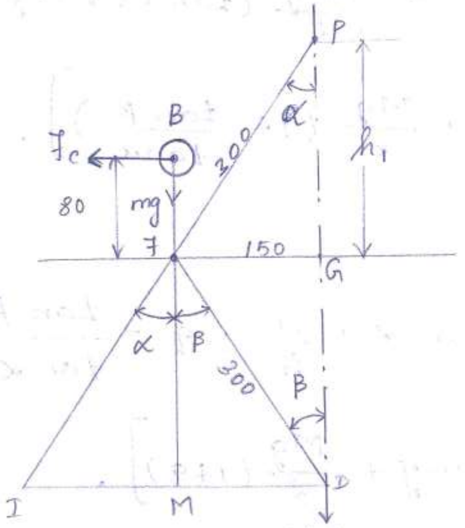
when $\alpha = \beta$ and $q = 1$,

③ ⇒ $\boxed{N^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\times\frac{895}{h}}$ — ④

Arms are of
same
ath.

Pbm

1) A proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are provided on the axis of the governor. The extension arms of the lower lines are each 80 mm long and $\parallel^{el}$ to the axis, When the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.



fig(a)

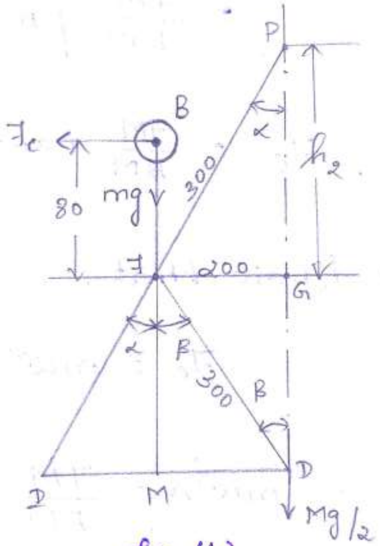


fig.(b)

Sn :-

$PF = DF = 300\ mm$; $BF = 80\ mm$; $m = 10\ kg$; $M = 100\ kg$.

$r_1 = 150\ mm$, $r_2 = 200\ mm$.

First of all we find the min and max speed of the governor.

Let,

$N_1 \rightarrow$ Min. speed when radius of rotation, $r_1 = FG = 150\ mm$.

$N_2 \rightarrow$ Max. speed when radius of rotation

From fig. (a), we find the height of the governor. Consider ΔFGP,

$$PF^2 = FG^2 + GP^2.$$

$$PF^2 = FG^2 + h_1^2.$$

$$\therefore h_1 = \sqrt{PF^2 - FG^2}$$

$$h_1 = \sqrt{300^2 - 150^2} = 260\ mm$$

$$\therefore h_1 = 0.26\ m$$

$$FM = GD = PG = 260\ mm = 0.26\ m$$

$$BM = BF + FM = 80 + 260 = 340\ mm = 0.34\ m.$$

W.K.T, $$N_1^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right) \times \frac{895}{h_1}$$

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26}$$

$$\therefore N_1 = 170\ rpm.$$

From fig. (b), consider ΔFGP,

$$PF^2 = FG^2 + PG^2.$$

$$\therefore h_2 = \sqrt{PF^2 - FG^2} = \sqrt{300^2 - 200^2}$$

$$\therefore h_2 = 0.224\ m$$

$$FM = GD = PG = 224\ mm = 0.224\ m.$$

$$BM = BF + FM = 80 + 224 = 304\ mm = 0.304\ m.$$

$$\therefore N_2^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_2}$$

$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224}$$

$$= 179.96$$

Range of speed:

$= N_2 - N_1$

$= 180 - 170$

$= 10$ rpm.

---

2) The following particulars to a Proell governor with open arms are given :

Length of the arms = 200mm, distance of pivot of arms from the axis of rotation = 40mm, length of extension of lower arms to which each ball is attached = 100mm, mass of each ball = 6 Kg, and mass of the central load = 150 Kg. If the radius of rotation of the balls is 180mm when the arms are inclined at an angle of 40° to the axis of rotation, find the equillibrium speed for the above configuration.

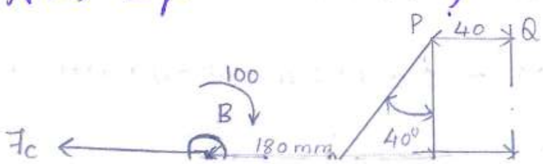
Gn :

$PF = DF = 200mm$, $PQ = DK = HG = 40mm$,

$BF = 100mm$, $m = 6$ kg, $M = 150$ kg, $r = JG = 180mm$
$= 0.18m$,

$\alpha = \beta = 40°$.

Let, $N \rightarrow$ Equillibrium speed.



From equillibrium position of governor as shown in figure, from ΔPFH,

$\cos 40° = \frac{PH}{PF}$.

$PH = \cos 40 \times 200 = 153.2mm$

$PH = 0.153 m$.

$|||^{ly}$, $\sin 40° = FH/PF \Rightarrow FH = \sin 40 \times 200$

$\therefore FH = 0.128 m$.

$JF = JG - HG - FGH$

$= 180 - 40 - 128.6$

$JF = 11.4 mm = 0.0114 m$.

From ΔBJF, using pythogoras thm,

$BF^2 = BJ^2 + JF^2$.

$BJ^2 = BF^2 - JF^2$.

$\therefore BJ = \sqrt{100^2 - 11.4^2}$

$\therefore BJ = 0.0994m$

W.K.T, $BM = BJ + JM = 0.994 + 0.153$

$BM = 0.252 mm$.

$IM = IN - MN$.

$= FH - JF$

$= 0.128 - 0.0114$

Taking moment about the instantaneous center I,

$$F_c \times BM = (mg \times IM) + \left(\frac{M \cdot g}{2} ID\right)$$

$$F_c \times 252.6 = (6 \times 9.81 \times 117.2) + \frac{150 \times 9.81}{2} \times 257.2$$

$$\therefore F_c = 776.4 \text{ N}.$$

W.K.T, centrifugal force, $F_c = m\omega^2 r$.

$$\Rightarrow 776.4 = 6\left(\frac{2\pi N}{60}\right)^2 (0.18)$$

$$= 0.012 N^2$$

$$\therefore N^2 = \frac{776.4}{0.012}$$

$$\therefore N = 254 \text{ rpm}.$$

3) A governor of proell type has each are 250mm long. The rivets of the upper and lower arm are 25mm from the axis. The central load acting on the sleeve has a mass of 3.2 kg. When the governor sleeve is in axial position, the extension link of the lower arm is vertical and the radius of the path of rotation of the masses is 175mm. The vertical height of the governor is 175 mm. The vertical height of the governor is 200 mm.

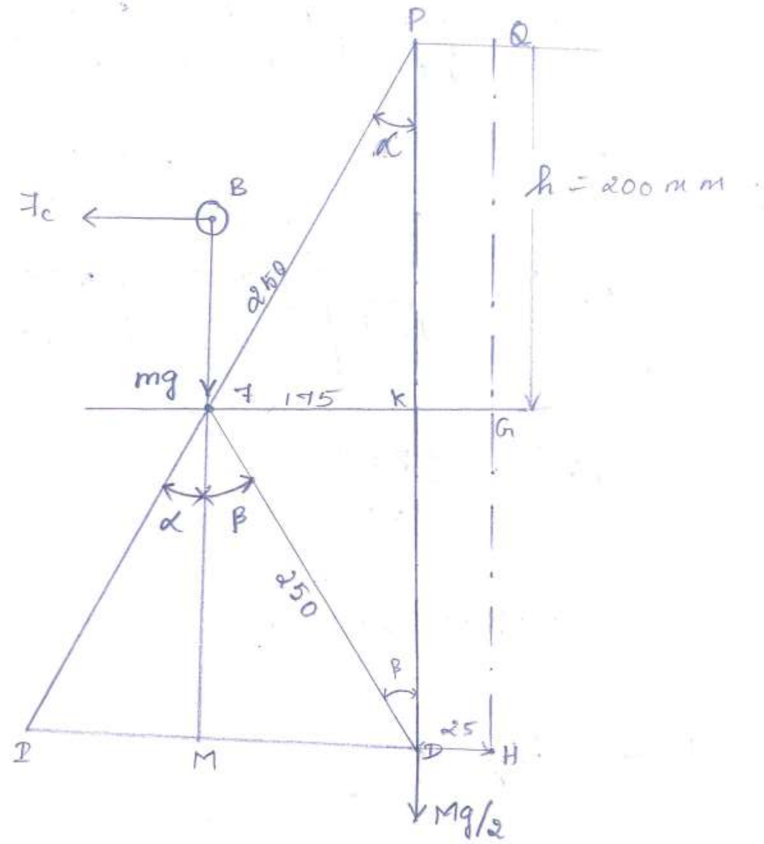
... is 160 rpm

when in axial position find,

1) Length of extension link.

2) Tension in the upper arm.

Soln:



Gn:

(i) Length of the extension link (BF)

$BF = DF = 250$ mm

$PQ = DH = KG = 25$ mm.

$M = 25$ kg

$m = 3.2$ kg.

$r = FG = 175$ mm.

$h = QG = PK = 200$ mm

$N = 160$ rpm.

Soln :

$FM = GH = QG = 200\ mm$.

W.K.T,

$$N^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\frac{895}{h}.$$

$(B = 2 \text{ or } q = 1)$.

$$160^2 = \frac{0.2}{BM}\left[\frac{3.2+25}{3.2}\right]\frac{895}{0.2}.$$

$$160^2 = \frac{7887}{BM}$$

$\therefore BM = 0.308\ m$.

From fig ;

$BF = BM - FM$.

$= 0.308 - 0.2$.

$BF = 0.108\ m$

(ii) Tension in the upper arm :

Let,

$T \rightarrow$ Tension in the upper arm.

In ΔPKF,

$$\cos\alpha = \frac{PK}{PF}$$

$$= \frac{200}{}$$

$\cos\alpha = 0.8$.

At equillibrium condition, summation of vertical force $= 0$,

$\Sigma V = 0$.

$$T_1 \cos\alpha = mg + \frac{M \times g}{2}.$$

$$T_1 = \frac{(3.2 \times 9.81) + \left(\frac{25 \times 9.8}{2}\right)}{0.8}$$

$T_1 = 192.5\ N$.

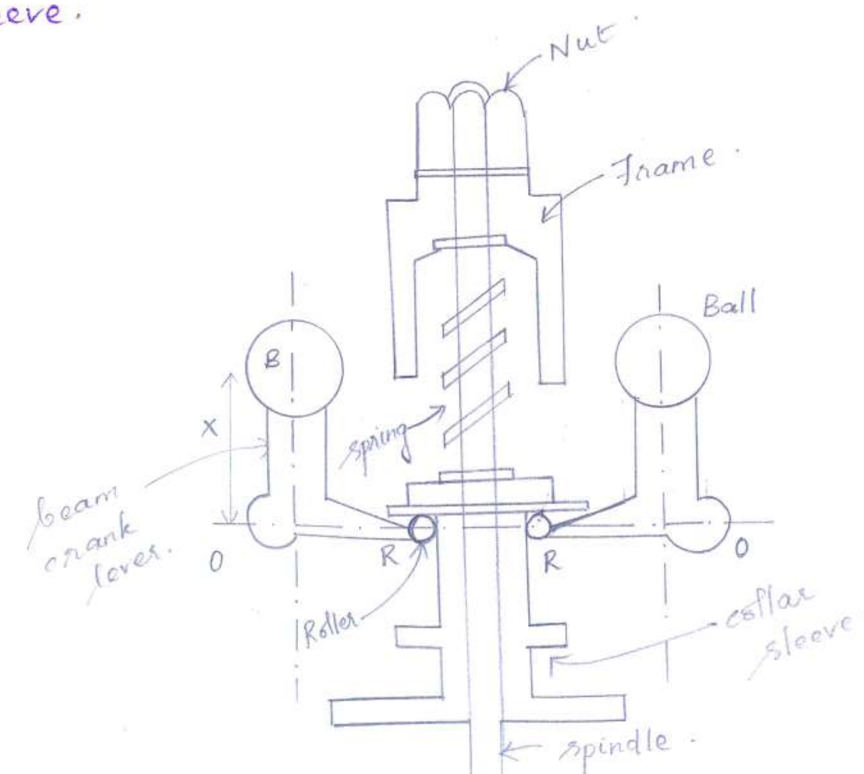
Result :

(i) Length of extension link, $BF = 0.108\ m$.

(ii) Tension in the upper arm, $T_1 = 192.5\ N$

Hartnell Governor:

A harnell governor is a spring loaded governor as shown in figure. It consists of two bell crank levers pivotted at the points 'O' to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



Let,

$m \longrightarrow$ mass of each ball in kg.

$M \longrightarrow$ mass of sleeve in kg.

$r_1 \longrightarrow$ Minimum radius of rotation in 'm'.

$r_2 \longrightarrow$ Maximum radius of rotation in 'm'

$\omega_1 \rightarrow$ angular speed of governor at minimum radius in rad/sec.

$\omega_2 \longrightarrow$ angular speed of governor at maximum radius in rad/sec.

$S_1 \longrightarrow$ spring force exerted on the sleeve at ω_1 in 'N'.

$S_2 \longrightarrow$ Spring force exerted on the sleeve at ω_2 in N.

$F_{c_1} \longrightarrow$ Centrifugal force at ω_1 in $N = m\omega_1^2 r$.

$F_{c_2} \longrightarrow$ centrifugal force at ω_2 in $N = m\omega_2^2 r$.

$s \longrightarrow$ stiffness of the spring

$x \longrightarrow$ length of the vertical or ball arm of the lever in 'm'.

$y \longrightarrow$ length of the horizontal or sleeve arm of the lever in 'm'.

$r \longrightarrow$ Distance of fulcrum 'O' from the governor axis or radius of rotation

Formula:

1) $h = (r_2 - r_1)\frac{y}{x}$.

2) Minimum position, $M.g + S_1 = 2F_{c_1} * \frac{x}{y}$

3) Maximum position, $M.g + S_2 = 2F_{c_2} * \frac{x}{y}$.

4) Stiffness of spring, $S = \frac{S_2 - S_1}{h}$.

1) Hartnell governor having a central sleeve spring and two right angled well cranked levers moves between 290 rpm and 310 r.p.m for a sleeve lift of 15mm. The sleeve arms and the ball arms are 80mm and 120 mm respectively. The levers are pivoted at 120mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine;

(i) Loads on the spring at the lowest and highest equilibrium speeds.

(ii) Stiffness of the spring.

Gn:

$N_1 = 290$ r.p.m.

$\omega_1 = \frac{2\pi N_1}{60} = 30.4$ rad/sec.

$N_2 = 310$ r.pm.

$\omega_2 = \frac{2\pi N_2}{60} = 32.5$ rad/sec.

$h = 15\,mm = 0.015\,m$.

$y = 80\,mm = 0.08\,m$.

$x = 120\,mm = 0.12\,m$.

$r = 120\,mm = 0.12\,m$.

$m = 2.5$ kg.

Soln:

(i) Loads on the spring at the lowest and highest speeds:

Let, $S_1 \rightarrow$ spring load at lowest equilibrium speed.

$S_2 \rightarrow$ spring load at highest equilibrium speed.

Since the ball arms are $\parallel^{el}$ to governor axis at the lowest equilibrium speed ($N_1 = 290$ rpm)

$\therefore\ r = r_1 = 120\,mm = 0.12\,m$.

W.K.T, centrifugal force at minimum speed,

$F_{c_1} = m\omega_1^2 r_1$

Let,

r_2 = radius of rotation at $N_2 = 310$ rpm.

W.K.T,

$$h = (r_2 - r_1)\, y/x.$$

$$hx = r_2 y - r_1 y.$$

$$r_2 = \frac{hx}{y} + \frac{r_1 y}{y}.$$

$$r_2 = r_1 + \frac{hx}{y}.$$

$$r_2 = 0.12 + \frac{(0.015)(0.12)}{0.08}$$

$$r_2 = 0.1425\,m.$$

Centrifugal force at maximum speed,

$$F_{c_2} = m\omega_2^2 r_2.$$

$$F_{c_2} = 25 \times (32.5)^2 \times 0.1425$$

$$F_{c_2} = 376\,N$$

Neglecting the obliquity effect of arms and moment due to the weight of walls

$\therefore$ Take, $M = 0$

$$\therefore \quad M.g + S_1 = 2F_{c_1} \times x/y$$

$$\therefore \quad S_1 = 2F_{c_1}(x/y).$$

$$= 2(277)\left(\frac{0.12}{0.08}\right).$$

$$S_1 = 831\,N$$

For maximum position,

$$M.g + S_2 = 2F_{c_2}(x/y).$$

here, $M = 0$.

$$\therefore \quad S_2 = 2(376)\left(\frac{0.12}{0.08}\right)$$

$$S_2 = 1128\,N.$$

(ii) Stiffness of the spring:

$$s = \frac{S_2 - S_1}{h}.$$

$$= \frac{1128 - 831}{15}$$

$$s = 19.8\,N/mm.$$

2) In a spring loaded hartnell type governer, the extreme radii of rotation of the balls are 80mm and 120mm. The ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 rpm and 420rpm Find, 1) The initial compression of the central spring.

2) The spring constant.

Soln:

$r_1 = 80mm$

$r_2 = 120 mm$.

$x = y$

$m = 2kg$

$N_1 = 400$ rpm.

$N_2 = 420$ rpm.

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi(400)}{60} = 41.9 \text{ rad/s}.$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi(420)}{60} = 44 \text{ rad/s}$$

$$F_{c_1} = m\omega_1^2 r_1 = 2(41.9)^2 0.08 = 281 N.$$

$$F_{c_2} = m\omega_2^2 r_2 = 2(44)^2 0.12 = 465 N.$$

S_1 = Spring force at the min speed.

S_2 = spring force at the max speed.

W.K.T,

Minimum position,

$$M.g + S_1 = 2F_{c_1} \times \frac{x}{y}.$$

$$S_1 = 2F_{c_1} = 2\times 281 = 562 N.$$

Maximum position,

$$M.g + S_2 = 2F_{c_2} \times \frac{x}{y}.$$

$$S_2 = 2\times 465$$

$$S_2 = 930 N.$$

W.K.T, Lift of the sleeve,

$$h = (r_2 - r_1)\, y/x$$

$$h = r_2 - r_1$$

$$h = 120 - 80.$$

$$h = 40 mm.$$

Stiffness of the spring, $s = \frac{S_2 - S_1}{h}$

$$= \frac{930 - 562}{40}$$

W.K.T,

1) Initial compression of the central spring,

$$= \frac{S_1}{S}$$

$$= \frac{562}{9.2}$$

$$= 61 \text{ mm}$$

2) Spring Constant :

We have calculated above spring constant or stiffness of the spring.

$$s = 9.2 \text{ N/mm}$$

unit

Sensitiveness of Governer :

The sensitiveness is defined as the ratio of difference between the maximum and minimum equillibrium speed to mean speed.

$$\text{Sensitivity of the governer} = \frac{N_2 - N_1}{N}.$$

$$= \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Isochronous governer :

A governer is said to be isochronou[s] when the equillibrium speed is constant for all radii of rotation of the balls with in the working range, of neglecting function. The isochronous is the stage of infinite sensitivity.

Let us consider the case of a porter governer running at speeds N_1 and N_2 rpm.

$$N_1^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1} \quad \text{—} ①$$

$$N_2^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2} \quad \text{—} ②$$

For isochronous 1° range of speed should be zero

$$N_2 - N_1 = 0.$$

Therefore from ① & ②, $h_1 = h_2$ which is impossible in case of a porter governer.

In harnell governer,

$$\frac{M.g + S_1}{M.g + S_2} = \frac{r_1}{r_2}.$$

(The isochronous governer is not of practical use because the sleeve will move to one of it's extreem position inmeadiatly.)

Effort and power of a governer:

The effort of governer is the mean force exerted at the sleeve for a given percentage change of speed. It may be noted that when the governer running steadily. There is no force at the sleeve. But when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort, varies uniformly opposes it's motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a maximum value of zero, while the governor moves into its new position of equillibrium.

The power of a governor is the work done at the sleeve for a given percentage change of speed. It is the product of the mean value of effort and the distance through which the sleeve motion, mathematically,

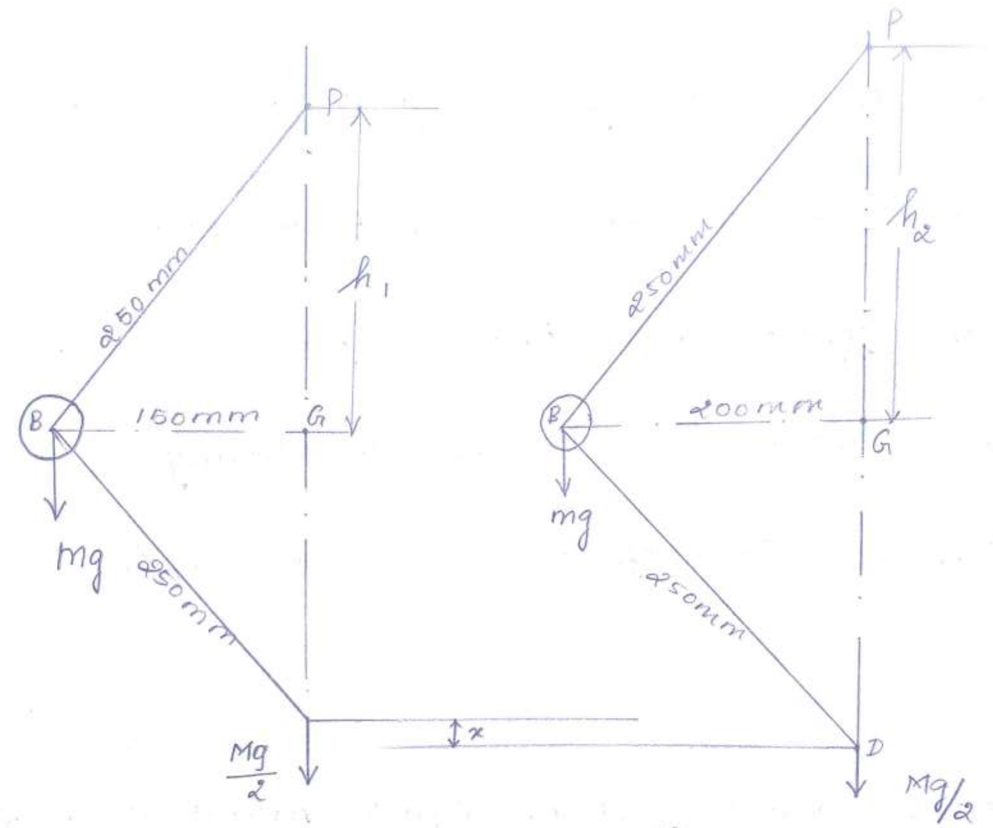
Power = Mean effort × lift of sleeves.

Pbm:

1) A porter governor has equal arms each 250mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of central load on the sleeve is 25 kg. the radius of rotation of the ball is 150mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases

(i) When the friction at the sleeve is neglected.

(ii) When the friction at the sleeve is equivalent



a) Minimum position (b) Maximum position

Sln:

$BP = BD = 250mm$, $m = 5 kg$, $M = 25 kg$,

$r_1 = 150 mm$, $r_2 = 200 mm$, $F = 10 N$.

(i) When the friction at the sleeve is neglected:

First of all let us find min & max speed of rotation.

From fig (a),

$$h_1 = PG = \sqrt{BP^2 - BG^2} = \sqrt{250^2 - 150^2}$$

$$h_1 = 0.2 m.$$

From fig (b)

$$h_2 = PG = \sqrt{BP^2 - BG^2} = \sqrt{250^2 - 200^2}$$

W.K.T, $N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+25}{5} \times \frac{895}{0.2}$

$\therefore N_1 = 164$ rpm.

$$N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+25}{5} \times \frac{895}{0.15}$$

$\therefore N_2 = 189$ rpm.

Range of speed, $N_2 - N_1 = 189 - 164 = 25$ rpm.

Sleeve lift, $x = 2(h_1 - h_2) = 2(200 - 150) = 0.1 m$

Governor effort:

$C \rightarrow$ Percentage increase in speed.

$CN_1 = N_2 - N_1 = 25$ rpm.

$\therefore C = 25/N_1 = 25/164 = 0.152$.

W.K.T, governor effort, $P = C(m+M)g = 0.152(5+25)9$

$\therefore P = 44.7 N$.

$\therefore$ Power of governor $= P \times x = 44.7 \times 0.1 = 4.47$ Nm

(ii) When the friction at the sleeve is taken into account:

$$N_1^2 = \frac{mg + (Mg - F)}{mg} \times \frac{895}{h_1}$$

$$= \frac{(5 \times 9.81) + ((25 \times 9.81) - (10))}{5 \times 9.81} \times \frac{895}{2}$$

$\therefore N_1 = 161$ rpm.

$$N_2^2 = \frac{m.g + (Mg + F)}{} \times 895$$

Sleeve lift, $x = 2(h_1 - h_2) = 0.1m$

Governor effort, $P = c(m+M)g$

$$c = \frac{N_2 - N_1}{N_1} = \frac{192-161}{161} = 0.193$$

$$\therefore P = 0.193(5+25)9.81 = 56.67N$$

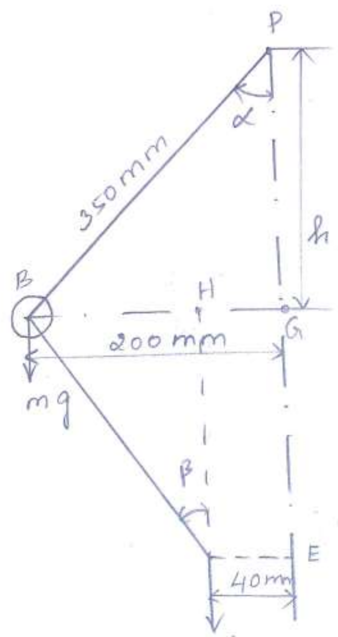
Power of governor $= P \times x = 56.67 \times 0.1 = 5.67$ Nm

2) The upper arms of a porter governor has lengths 350mm and are pivoted on the axis of rotation. The lower arm has lengths 300mm and are attached to sleeve at 40mm from axis. Each ball has a mass of 4 kg and mass on the sleeve is 45 kg. Determine the equillibrium speed for a radius of rotation 200mm and also find the effort and power of the governor 1% speed change.

Gn:

$PB = 350mm = 0.35m$; $BD = 300mm = 0.3m$, $DE = 0.04m$

$m = 4$ kg, $M = 45$ kg; $r = BG = 200mm = 0.2m$.



Equillibrium speed,

Let, N - equillibrium speed.

The equillibrium ~~speed~~ position is shown in the fig. from geometry of the fig,

$$h = PG = \sqrt{PB^2 - BG^2}$$

$$= \sqrt{0.35^2 - 0.2^2} = 0.287 m.$$

$$\tan\alpha = \frac{BG}{PG} = \frac{0.2}{0.287} = 0.697.$$

$$BH = BG - HG = 0.2 - 0.04 = 0.16m$$

$(\because HG = DE)$.

$$DH = \sqrt{BD^2 - BH^2}.$$

$$= \sqrt{0.3^2 - 0.16^2} = 0.254m.$$

$$\tan\beta = \frac{BH}{DH} = \frac{0.16}{0.254} = 0.63.$$

$$q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.697} = 0.904.$$

We know that,

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} = \frac{4 + \frac{45}{2}(1+0.904)}{4} \times \frac{895}{0.287}$$

$N^2 = 36517 \Rightarrow N = 191$ rpm.

Effort of the governor:

$$P = c\left[\frac{2m}{1+q} + M\right] \cdot g = 0.01\left[\frac{2\times4}{1+0.904} + 45\right] \times 9.81.$$

$P = 4.8$ N.

Power of the governor:

$$= \frac{4c^2}{1+2c}\left[m + \frac{M}{2}(1+q)\right] g.h.$$

$$= \frac{4(0.01)^2}{1+2(0.01)}\left[4 + \frac{45}{2}(1+0.904)\right] 9.81 \times 0.287$$

$= 0.052$ Nm

Unit - I. Force Analysis.

Part - I / Dynamic force analyis

Inertia :

The property of matter offering resistance to any change of it's state of rest or of uniform motion in a straight line is known as inertia.

Inertia force :

The inertia of a body opposes this external force applied (F) and it is known as inertia force.

$$\boxed{\begin{aligned}\text{Inertia force} &= -\text{external (accelerating) force} \\ &= -m \cdot a\end{aligned}} \quad ①$$

Inertia torque :

The inertia of the body opposes this exertal torque applied (T) and it is known as inertia torque.

$$\boxed{\text{Inertia torque} = -\text{externally applied torque}} \quad ②$$

D' Alembert's principle :

It states that, the inertia forces and torques, and the external forces and torques, acting on a body together results in statical equillibrium.

The eqns ① and ② can also be written as

$$F + (-ma) = 0 \quad ③$$

$$T + (-I\alpha) = 0 \quad ④$$

The above eqns ③ & ④ are known as D'Alembert's principle.

eqns ③ & ④ can also be written as,

$$\left.\begin{aligned}\Sigma F &= 0 \\ \Sigma M &= 0\end{aligned}\right\} \quad ⑤.$$

Application of D' Alembert's principle :

This principle is used to reduce a dynamic analysis problem into an equivalent problem of static equillibrium.

Dynamic Analysis of reciprocating engines:

i) Velocity and Acceleration of reciprocating parts in engine:

The velocity and acceleration of various parts of reciprocating mechanism can be determined both analytically and graphically. The various methods used are

(i) Analytical Method.

(ii) Graphical Method.

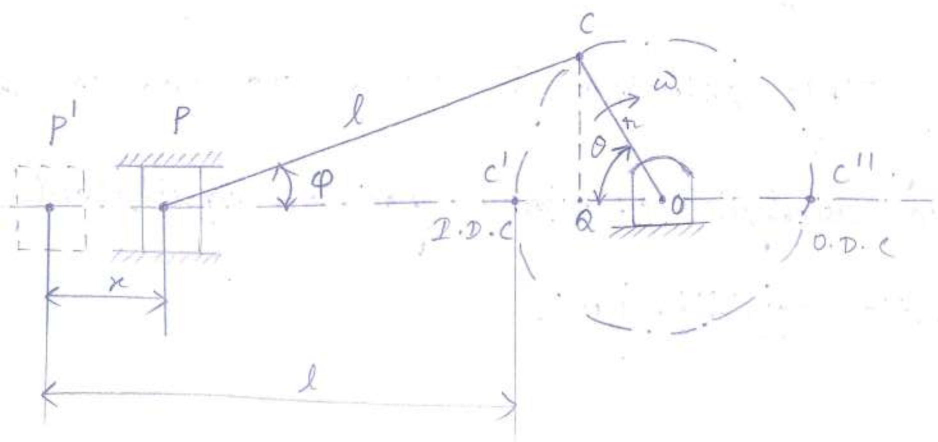
a) Klein's construction.

b) Ritterhaus's construction.

c) Benett's construction.

Analytical Method:

fig: Reciprocating Engine Mechanism.



Let,

$r \rightarrow$ crank radius.

$l \rightarrow$ length of connecting rod.

$\theta \rightarrow$ Angle made crank with I.D.C.

$\phi \rightarrow$ Inclination of connecting rod to the line of stroke PO.

$n = \frac{l}{r}$ = Ratio of length of connecting rod to the radius of crank, also known as 'obliquity ratio'.

Forces on the reciprocating parts of an engine, neglecting the weight of the connecting rod:

A] To find the net load on the piston (F_L):

(i) For single-cylinder single acting engine:

Let,

$P \rightarrow$ Net pressure of steam(or)gas on the piston in N/m^2.

$D \rightarrow$ Diameter of the piston in 'm'.

Then, Net load on the piston is given by,

F_L = Pressure x Area.

$$F_L = P \times \frac{\pi}{4} D^2.$$

Unit-5

Part-2 / GYROSCOPE:

Gyroscopic couple (torque):

Whenever a rotating body changes its axis of rotation, a couple (torque) is applied on the rotating body. This couple (torque) is known as gyroscopic couple (torque)

Practical examples of gyroscopic motion:

1) When automobile vehicles (motorcycles, cars) move in a curved path, gyroscopic couple acts on spinning parts such as crank shaft, flywheel, clutch, transmission gears, propellar shaft and wheels.

2) When an aeroplane takes a turn, gyroscopic effect influence the engine parts

3) Similar gyroscopic effects are realised when locomotives taking a turn.

Review and Summary:

* Gyroscopic acceleration, $\alpha_c = \omega \cdot \omega_p$.

... velocity of the disc.

* Gyroscopic couple, $C = I\omega\omega_p$.

where, $I = mk^2$ = Mass moment of inertia.

$\omega_p = V/R$ = Angular velocity of precession.

$V \rightarrow$ Linear velocity.

$R \rightarrow$ Radius of curvature.

* The effects of gyroscopic couple on an aeroplane taking a turn are tabulated below.

S.No	View Point	Direction of propellar rotation	Turn	Effect.
1.	Rear end	clockwise	Left NR	Nose raised, Tail depressed.
2.	"	"	Right ND	Nose depressed, Tail raised.
3.	"	Anticlockwise	Left ND	Nose depressed, Tail raised.
4.	"	"	Right NR	Nose raised, Tail depressed.
5.	Front end	"	Left NR	Nose raised, Tail depressed.
6.	"	"	Right ND	Nose depressed, Tail raised
7.	"	Clockwise	Left ND	Nose depressed, Tail raised
8.			NR	Nose raised

Table: Gyrospic effect on ship during steering.

S.No	View Point	Direction of Propeller rotation	Turn	Effect
1	Steern (rear)	clockwise	Left BR	Bow raised, steer depressed.
2	"	"	Right BD	Bow depressed, steer raised.
3	"	Anticlockwise	Left BD	"
4	"	"	Right BR	Bow raised, steer depressed
5	Bow	"	Left BR	"
6	"	"	Right BD	Bow depressed, steer raised.
7	"	Clockwise	Left BD	"
8	"	"	Right. BR	Bow raised, steer depressed.

Gyroscopic chart for ship during pitching:

S.No	Pitching	View Point	Direction of rotar rotation	Effect.
1.	Upward	Stern (rear)	CW S	ship turns towards star-board side [illegible]
2.	"	"	ACW P	ship turns towards port side [illegible]
3.	"	Bow	CW P	ship turns towards port side [illegible]
4.	"	"	ACW S	ship turns towards star-board side [illegible]
5.	Downward	Stern (rear)	CW P	ship turns towards port side. [illegible]
6.	"	"	ACW S	ship turns towards star-board side [illegible]
7.	"	Bow	CW S	ship turns towards star-board side [illegible]
8.	"	"	ACW P	ship turns towards port side. [illegible]

* Gyroscopic effect on pitching:

(i) Maximum gyroscopic couple, $C_{max} = I\omega\omega_{p(max)}$

where, $\omega_{p(max)} = \varphi \times \omega_o = \varphi \times \frac{2\pi}{t_p}$.

(ii) Maximum angular acceleration during pitching

$$\alpha_{max} = \varphi . \omega_o^2 .$$

* Stability of a 4-wheel automobile moving in a curved path:

1) Reaction due to the weight of the ~~ha~~ vehicle

$$= \frac{W}{4} = \frac{mg}{4}.$$

2) Reaction due to gyroscopic couple,

$$C = \omega_W . \omega_p \, (4 I_W \pm G I_E)$$

Vertical reaction at each of the outer and inner ~~kills~~, wheels,

$$\frac{P}{2} = \frac{C}{2x}.$$

3) Reaction due to centrifugal effect.

$$C = \left(\frac{mv^2}{R}\right) h$$

Vertical reaction at each of the outer and inner wheels,

$$\frac{Q}{2} = \frac{mv^2 h}{2Rx}.$$

4) For stability for 4-wheel vehicle,

$$\frac{W}{4} \geq \frac{P}{2} + \frac{Q}{2}$$

* <u>Stability of 2-wheel vehicle;</u>

Gyroscopic couple, $C_1 = \frac{v^2}{r_W}(2I_W \pm G.I_E)\cos\theta$

Centrifugal couple, $C_2 = \left(\frac{mv^2}{R}\right)h\cos\theta.$

Total overturning couple, $C_\theta = \frac{v^2}{R}\left[\frac{2I_W + G.I_E}{r_W} + mh\right]\cos\theta.$

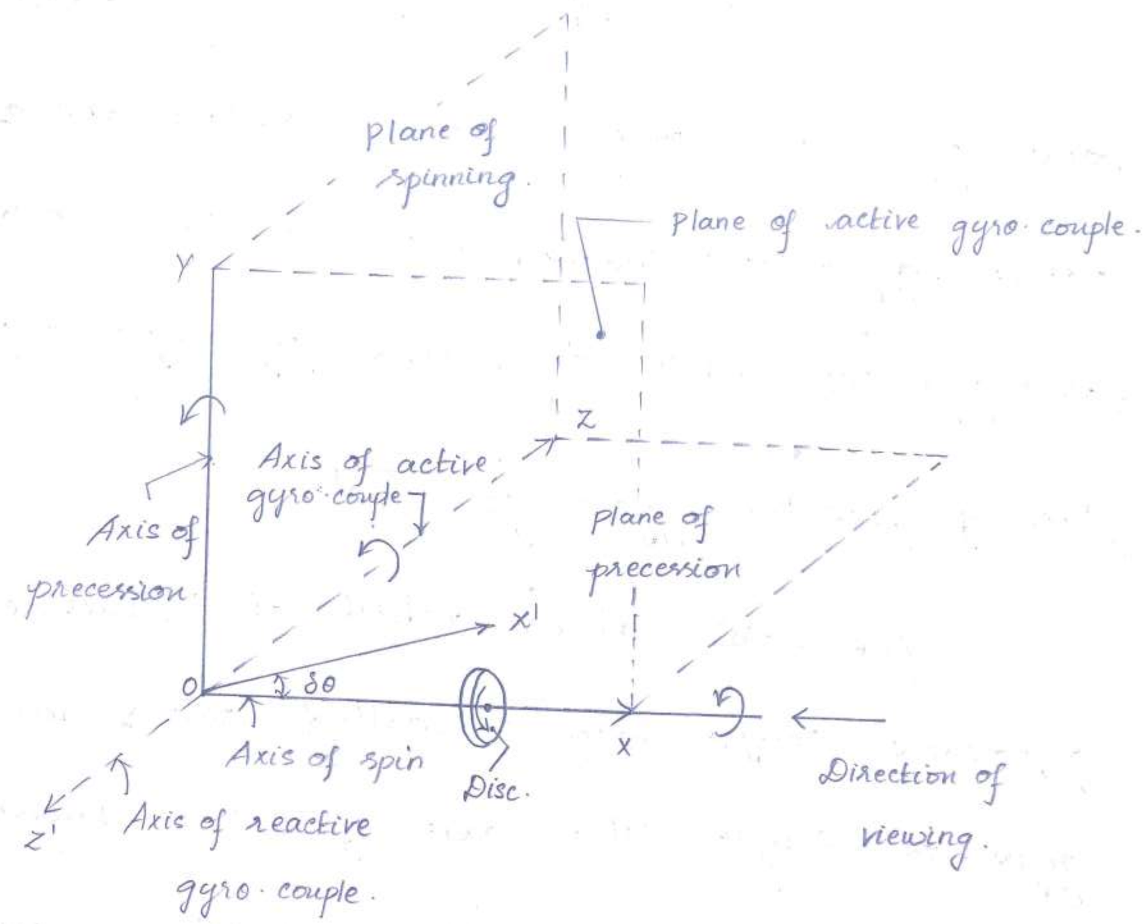
$$C_\theta = C_1 + C_2$$

* For stability of the two wheel vehicle, the overturning couple must be equal to the balancing couple.

$$\frac{v^2}{R}\left[\frac{2I_W + GI_E}{r_W} + mh\right]\cos\theta = mgh\sin\theta.$$

Balancing couple $= mgh\sin\theta$

Gyroscopic Couple.



(i) Since the plane in which a disc is rotating (spinning) $\parallel^{el}$ to the plane YOZ, therefore plane YOZ is known as the <u>plane of spinning.</u>

(ii) The plane XOZ is the horizontal plane. OY axis is known as the <u>axis of precession</u> and plane XOZ is known as <u>plane of precession</u>. The angular motion of the axis of spin about the axis of precession is known as <u>precessional angular motion</u>

Axis of Active gyroscopic couple:

The axis about which the active ~~axis~~ gyroscopic couple acts is called axis of active gyroscopic couple. The axis OZ is known as gyroscopic axis or active gyroscopic couple.

Axis of reactive gyroscopic couple:

The axis about which the reactive gyroscopic couple acts is called axis of reactive gyroscopic couple. The axis of OZ' is known as gyroscopic axis or reactive gyroscopic couple

Plane of active gyroscopic couple:

The plane which is $\perp^r$ to the line representing the change in angular momentum is known as plane of active gyroscopic couple. Plane XOY is known as plane of active gyroscopic couple.

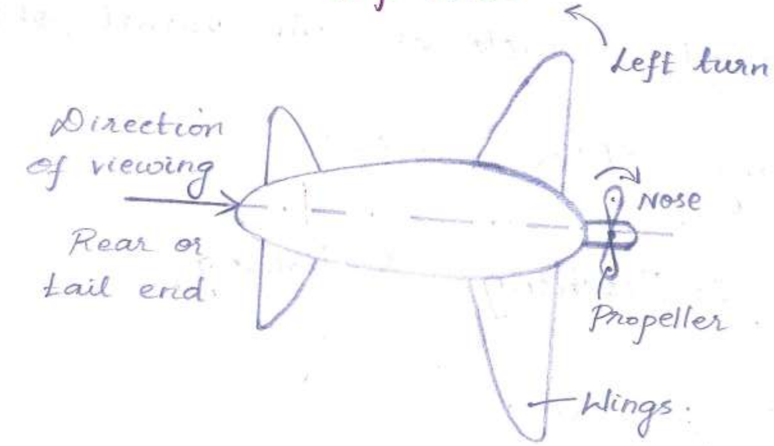
Plane of reactive gyroscopic couple:

The plane which is $\perp^r$ to the line representing the change in angular momentum but 180° opposite to the plane of active gyroscopic couple is known as plane of reactive gyroscopic couple.

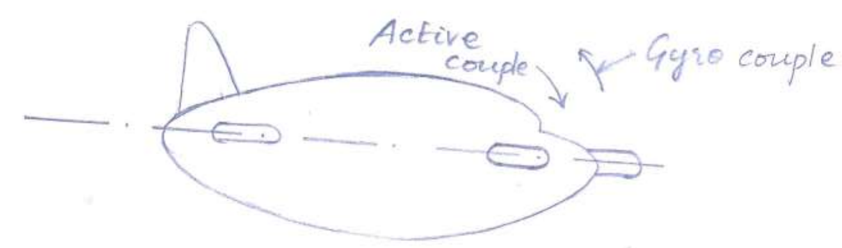
Effect of gyroscopic couple on an aeroplane:

The top and front view of an aeroplan is shown in the figure. Let engine or propeller rotates in clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.

Top View



Front view.

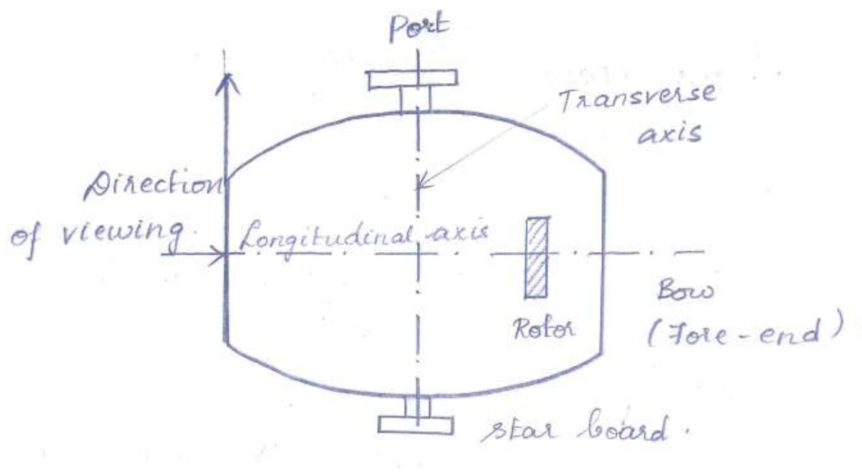


Terms used in a Naval ship:

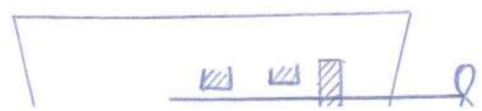
The top and front views of a naval ship are shown in figure. The fore end of the ship is called bow and rear end is known as a stern or aft. The left hand and right hand sides of the ship, when viewed from the stern are called port and star-board respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

1. Steering 2. Pitching 3. Rolling.

Top View.



Front view.



Effect of gyroscopic couple on a naval ship during shearing:

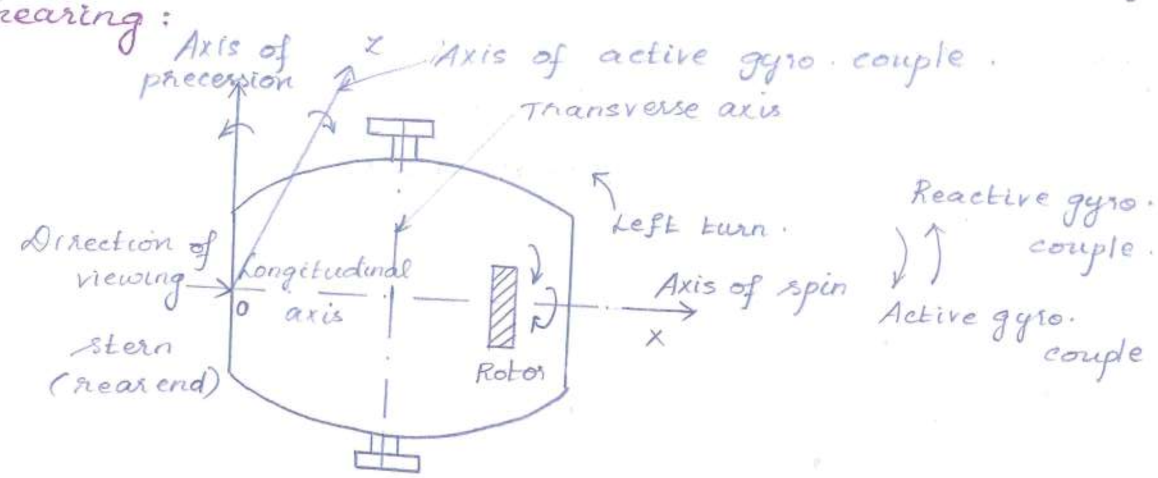
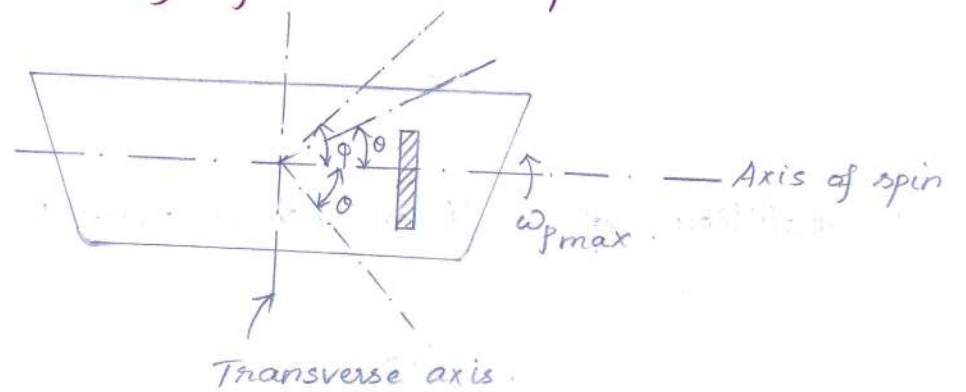


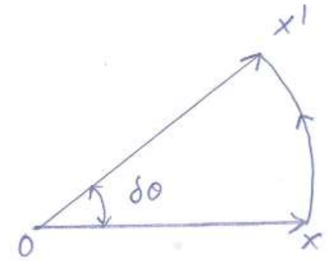
fig: (Naval ship taking a left turn).

Effect of gyroscopic couple on a naval ship during pitching:

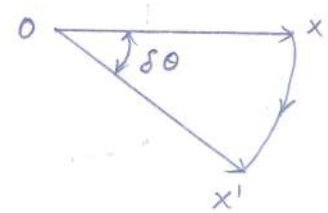
Pitching of a naval ship.



Pitching upward

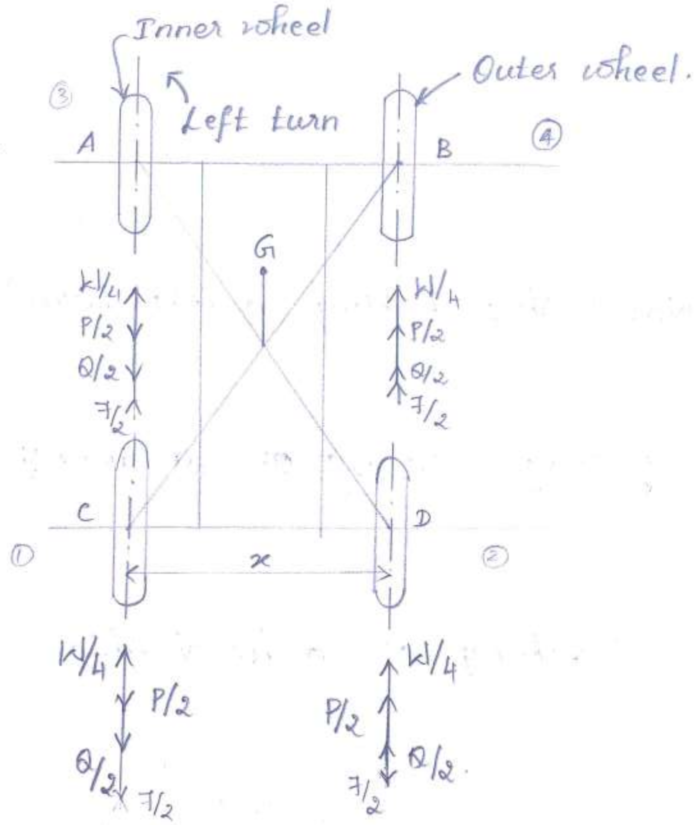


Pitching downward.

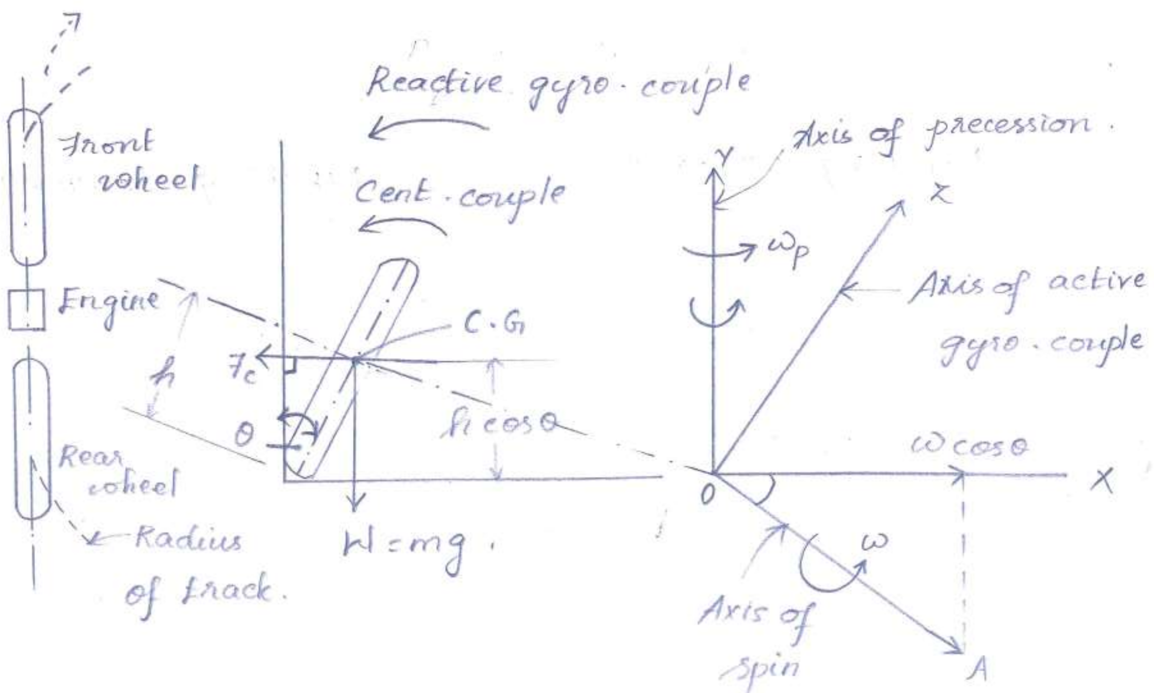


Stability of a four wheeler drive in a curved path:

Four wheeler drive.



Stability of a Two wheel vehicle taking turn:



Gyroscopic effect on rolling:

Rolling is the side way motion of the ship about it's longitudinal axis. During rolling the axis of rolling (spin) and that of rotar of turbine are generally same. There is no precession of axis of spin and therefore there is no gyroscopic effect. When the ship rolls

Problems on steering of ships:

1) A ship sails at a speed of 125 km/hr. The mass of its turbine rotar is 600 kg having a radius of gyration of 0.6 m. It rotates at 1600 rpm in a clockwise direction when looking from it's stern. When the ship steers to the left in a radius of curvature of 110 m, what would be the gyroscopic couple acting on the ship and what would be it's effect. In

Gn:

$V = 125$ km/hr.

$= \frac{125 \times 1000}{3600}$

$m = 600$ kg

$N = 1600$ rpm.

$k = 0.5$ m

$R = 110$ m.

Soln:

(i) Gyroscopic couple: (c)

Mass moment of Inertia, $I = mk^2$.

$= 600(0.6)^2$.

$I = 216 \ kg m^2$.

Angular velocity of rotar $\omega = \frac{2\pi N}{60}$.

$= 167.55$ rad/s.

Angular velocity of Precession, $\omega_p = \frac{V}{R}$

$= \frac{34.72}{110}$

$\omega_p = 0.316$ rad/s.

$\therefore$ Gyroscopic couple, $c = I\omega\omega_p$.

$= 216 \times 167.55 \times 0.316$.

$= 11436.3$ Nm.

(ii) Effect of gyroscopic:

S.No	View point	Direction of rotar rotation	Turn	Effect.
1.	Stern	Clockwise	Left	Bow raised, steer depressed.

2) A high speed ship is driven by a turbine rotar which has a moment of inertia of 20 kg m² and is running at 3000 rpm in clockwise direction when viewed from the bow. The ship is speeding at 72 km/hr taking a right turn around a curve of 600 m radius. Determine the gyroscopic couple applied to the ship and it's effects.

Gn:

$I = 20 \ kg m^2$.

$N = 3000$ rpm.

$V = 72$ km/hr $= \frac{72 \times 1000}{3600} = 20$ m/s.

$R = 600$ m.

Soln:

(i) Gyroscopic couple: (c):

Mass moment of inertia, $I = \cancel{I}\ 20 \ kg m^2$.

Angular velocity of rotar, $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi(3000)}{60}$

$= 314.16$ rad/s.

Angular velocity of precission, $\omega_p = \frac{V}{R}$.

$$= \frac{20}{600}$$

$\omega_p = 0.033$

$\therefore$ Gyroscopic couple, $C = I \omega \omega_p$

$= (20)(314.16)(0.033)$

$C = 207.345$ Nm.

Gyroscopic Effect:

S.NO	View Point	Direction of rotar rotation	Turn	Effect.
8)	Bow	CW	Right	Bow raised, steer depressed.

3) Each wheel of a steamer have a mass 1600 kg and a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km/hr and running at a speed of 90 rpm. Find the magnitude and effect of gyroscopic couple acting on the steamer.

Gn:

$m = 1600$ kg.

$k = 1.2$ m.

$R = 160$ m.

$V = 24$ km/hr $= \frac{24 \times 1000}{3600} = 6.67$ m/s

$N = 90$ rpm.

Soln: (i) Magnitude of gyroscopic couple (C):

Mass moment of inertia, $I = mk^2$.

$= 1600(1.2)^2$.

$I = 2304$ Kgm².

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi(90)}{60}$

$\omega = 9.425$ rad/s

$\omega_p = \frac{V}{R}$

$= \frac{6.67}{160}$

$\omega_p = 0.042$ rad/s.

$\therefore$ Gyroscopic couple, $C = I\omega\omega_p$

$= (2304)(9.425)(0.042)$

$= 905.25$ Nm.

(ii) Gyroscopic effect:

In our Pbm. the view point (ie) direction of viewing and the sense of rotation of the rotar are not specified. As discussed in previous table the possible gyroscopic effects during steering are summarized as shown below.

S.No	View Point	Direction of propellar rotation	Turn	Effect.
1.	Stern	Clockwise	Left	Bow raised, steer depressed.
2.	"	CW	Right	Bow depressed, steer raised
3.	"	Anticlockwise	Left	"
4.	"	ACW	right	Bow raised, steer depressed
5.	Bow	ACW	Left	"
6.	"	ACW	right	Bow depressed, steer raised.
7.	"	CW	Left	"
8.	"	CW	right	Bow raised, steer depressed.

Problems on the pitching of ships:

1) The rotar of a turbine in a boat with it's axis along the longitudinal axis of the boat makes 1500 rpm CW when viewed from the stern. The rotar has a mass of 750 kg and a radius of rotation of 300mm. If at an instant, the boat pitches in the longitudinal vertical plane so that bow raises from the horizontal plane with an angular velocity of 1 rad/sec, determine the torque acting in the boat and the direction in which it tends to turn the boat at the instant.

Gn:

$N = 1500$ rpm

$m = 750$ kg

$K = 300$ mm $= 0.3$ m.

$\omega_p = 1$ rad/sec.

Soln:

(i) Gyroscopic couple:

Mass moment of inertia, $I = mK^2$.

$= 750 \times 0.3^2$

$= 67.5$ kg m^2.

$\omega = \frac{2\pi N}{60} = \frac{2\pi(1500)}{60} = 157.08$ rad/sec

$\omega_p = 1$ rad/s (Gn)

$\therefore$ Gyroscopic couple, $C = I\omega\omega_p$

$= 67.5 \times 157.08 \times 1$.

$= 10602.9$ Nm.

$C = 10.602$ KNm.

ii) Gyroscopic effect:

When the rotar rotates clockwise looking from stern and the ship pitches upwards (bow raises) is to turn the ship towards star-board side

S.No	View point	Direction of rotar rotation	Effect.	Pitching
1.	stern	clockwise	ship turns towards star-board side	upward.

2) The marine turbine rotar of inertia 700 Kg m^2 rotates at 2800 rpm clockwise when viewed from ~~left~~ stern. If ship pitches with angular SHM with a period of 6 sec and amplitude of 0.2 radian. Find (i) Maximum angular velocity of rotar.

(ii) Maximum gyroscopic couple.

(iii) Maximum gyroscopic effect as the bow dips.

Gn data:

$I = 700 \text{ Kg m}^2$.

$N = 2800$ rpm.

$t_p = 6$ sec.

$\phi = 0.2$ rad.

Soln:

i) Angular velocity of rotar: $(\omega_p max)$

$$\text{Angular velocity}, \omega = \frac{2\pi N}{60} = \frac{2\pi(2800)}{60}$$

Angular velocity of SHM, $\omega_0 = \frac{2\pi}{t_p}$

$$= \frac{2\pi}{6}$$

$$\omega_0 = 1.047 \text{ rad/s}$$

$\therefore \omega_p max = \phi\omega_0 = 0.2 \times 1.047 = 0.2094$ ra

ii) Maximum gyroscopic couple (C_{max}):

$$C_{max} = I\omega \cdot \omega_p max.$$

$$= 700 \times 293.2 \times 0.2094.$$

$$C_{max} = 42.98 \text{ KNm}.$$

(iii) Gyroscopic effect as the bow dips:

S.No	Pitching	View Point	Direction of rotar rotation	Effect.
5.	Downward	stern	CW	ship turns towards port side

3) The turbine rotar of a ship has a mass of 20 tonnes and a radius of gyration of 0.75m. It's speed is 2000 rpm. The ship pitches 6° abov and below the horizontal position. one complet oscillation takes 18 sec and their motion is SHM Determine

(a) The maximum couple tending to shear the holding down bolts of the turbine.

(b) The maximum angular acceleration of the

c) The direction to which the bow will tend to turn while rising, if the rotation of the rotor is clockwise when looking from rear.

Gn:

$m = 20$ tonnes

$m = 20 \times 10^3$ kg

$k = 0.75$ m.

$N = 2000$ rpm.

$\phi = 6^\circ = \frac{\pi}{180} \times 6 = 0.105$ rad

$t_p = 18$ sec.

Soln:

(a) Max. Couple (C_{max}):

$$I = mk^2 = 20 \times 10^3 \times 0.75^2 = 11250 \text{ Kgm}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(2000)}{60} = 209.44 \text{ rad/sec}$$

$$\omega_0 = \frac{2\pi}{t_p} = \frac{2\pi}{18} = 0.349 \text{ rad/sec.}$$

$$\therefore \omega_{Pmax} = \phi\,\omega_0 = 0.105 \times 0.349 = 0.0366 \text{ rad/sec}$$

$$\therefore C_{max} = I\,\omega\,\omega_{Pmax}$$

$$= 11250 \times 209.44 \times 0.0366$$

$$= 86359.24 \text{ Nm}$$

$$C_{max} = 86.36 \text{ KNm.}$$

b) Max. angular acceleration of the ship during pitching (α_{max}):

$$\alpha_{max} = \phi\,\omega_0^2.$$

$$= 0.105 \times 0.349^2.$$

$$\alpha_{max} = 0.0128 \text{ rad/s}^2.$$

c) Gyroscopic Effect:

S.No	Pitching	View Point	Direction of rotar rotation	Effect.
1.	Upward	Stern (rear)	CW	Ship turns towards star-board side.

Problems on steering, pitching and rolling of ships:

1) The rotar of the turbine of a ship has a mass of 2400 kg and rotates at a speed of 3100 rpm anticlockwise when viewed from the stern. The rotar has radius of gyration of 0.4 m. Determine the gyroscopic couple and it's effect when

(i) The ship stears to the left in a curve of 78 m radius at a speed of 14 knots (1 knot = 1860 m/hr)

(ii) The ship pitches 5° above and below the normal position and the bow is descending with it's maximum velocity. The pitching motion is simple harmonic with a periodic time of 35 seconds.

angular velocity is 0.03 rad/sec clockwise when viewed from stern.

Gn:

$m = 2400$ kg

$N = 3100$ rpm.

$k = 0.4$ m.

(i) When the ship steers to the left:

here, $v = 14$ knots.

$= 14 \times 1860$

$= 26040$ m/hr.

$$v = \frac{26040}{3600}$$

$v = 7.233$ m/s.

$R = 78$ m.

Gyroscopic couple:

Angular velocity of precession,

$$\omega_p = \frac{V}{R} = \frac{7.233}{78}$$

$\omega_p = 0.0927$ rad/s.

Gyroscopic couple, $C = I\omega\omega_p$.

$I = mk^2 = 384$ kgm^2.

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi(3100)}{}$$

$C = I\omega\omega_p$.

$= 384 \times 324.63 \times 0.0927$.

$C = 11560.16$ Nm

Gyroscopic effect:

Sno	View Point	Direction of propellar rotation	Turn	Effect.
3.	stern	Anticlockwise	Left	Bow depressed, steer raised.

ii) When the ship pitches with the bow descending:

here, $\phi = 5^\circ = 5 \times \frac{\pi}{180} = 0.0873$ rad.

$t_p = 35$ sec.

Gyroscopic couple:

Angular velocity of SHM, $\omega_0 = \frac{2\pi}{t_p} = \frac{2\pi}{35}$

$\omega_0 = 0.1795$ rad/s.

Maximum angular velocity of precession, $\omega_p = \phi\omega_0$.

$= 0.0873 \times 0.1795$

$\omega_{p_{max}} = 0.0157$ rad/s.

$\therefore$ Maximum gyroscopic couple, $C_{max} = I\omega\omega_{p_{max}}$.

$= 384 \times 324.63 \times 0.0157$.

$= 1953.64$ Nm.

Gyroscopic effect for pitching:

S.no	View point	Pitching	Direction of rotar rotation	Effect.
6.	stern	downwards ~~Upwards~~	Anticlockwise	ship turn towards star-board side.

(iii) When the ship rolls:

here, $\omega_p = 0.03$ rad/s.

Gyroscopic couple, $C = I\omega\omega_p$.

$= 384 \times 324.63 \times 0.03$.

$C = 3739.74$ Nm.

Gyroscopic effect:

During rolling the axis of spin is always parallel to the axis of precession for all positions, there is no gyroscopic effect on the ship.

(2m)

Terms used in Gyroscopic couple of ship:

1) Bow:

It is the fore or front end of ship

2) Stern (or) aft:

It is the rear end of ship

3) Port:

It is the left hand side of the ship when looking from rear end.

4) Star-board:

It is the right hand side of the ship when looking from rear end

Types of movements:

1) Steering:

It is the turning of a ship in a curve either to the right or to the left hand side while the ship move forward.

2) Pitching:

It is the upward or downward angular moment of the ship in a vertical plane about it's transreverse axis from the horizontal position.

3) Rolling:

It is the sideway motion of the ship about it's longitudinal axis.

Gyroscopic effect on aeroplanes:

NOTE:

Let the engine / propeller rotates in the clockwise direction when seen from the rear or tail end of the aeroplane and the aeroplane takes a turn to the left

Problems on gyroscopic effect on aeroplanes:

1) An aeroplane makes a complete half circle 60 m radius to the left when flying at 200 km/hr. The rotar engine and the propeller of the plane weights 4000 N with a radius of gyration 30 cm and the engine runs at 2250 rpm clockwise, when viewed from rear. Find the gyroscopic couple on the aircraft and states it's effect on it.

Gn:

$R = 60\ m$

$V = 200\ km/hr = \dfrac{200 \times 1000}{3600} = 55.55\ m/s.$

$W = 4000\ N$

(or)

$m = \dfrac{4000}{9.81} = 407.75\ kg.$

$k = 30\ cm = 0.3\ m.$

$N = 2250\ rpm.$

From NOTE, engine or propeller rotates CW direction. aeroplane takes left turn, when viewed from rear.

Soln:

Gyroscopic couple:

We know that mass moment of inertia of the engine or propeller, $I = mk^2$.

$= 407.75\ (0.3)^2.$

$= 36.697$

$= 36.7\ kgm^2.$

Angular velocity, $\omega = \dfrac{2\pi N}{60}$

$= \dfrac{2\pi \times 2250}{60}$

$= 235.62\ rad/sec$

Angular velocity of precession, $\omega_p = \dfrac{V}{R} = \dfrac{55.55}{60}$

$\omega_p = 0.926\ rad/s.$

We know that gyroscopic couple acting on the aircraft,

$C = I\omega\omega_p.$

$= 36.7 \times 235.62 \times 0.926.$

$C = 8005.9\ Nm.$

Gyroscopic effect: (CW, rear, left) S.No:1.

S.No	View point	Direction of Propeller rotation	Turn	Effect
1.	Rear End	clockwise	Left	Nose raised, Tail depressed.

2) The rotor of a turbojet engine has a mass of 210 kg and a radius of gyration 250 mm. The engine rotates at a speed of 9500 rpm in clockwise direction if viewed from the front of the aeroplane. The aeroplane while flying at 975 kmph turns with a radius of 2.25 km to the right. Compute the gyroscopic movement exerted by the rotor on the plane structure. (Also determine whether the nose of plane tends to rise or fall when the plane turns) Gyroscopic effect

Gn:

$m = 210$ kg

$k = 250$ mm $= 0.250$ m.

$N = 9500$ rpm.

$v = 975$ kmph $= 975$ km/hr $= \frac{975 \times 1000}{3600}$

Soln:

Gyroscopic couple:

W.K.T, mass moment of inertia, $I = mk^2$.

$= 210 \times (0.250)^2$.

$= 13.125$ kg m^2.

Angular velocity, $\omega = \frac{2\pi N}{60}$

$= \frac{2\pi (9500)}{60}$

$= 994.84$ rad/s.

Angular velocity of precession, $\omega_p = \frac{V}{R}$

$= \frac{270.83}{2.25 \times 10^3}$

$= 0.1204$ rad/s

Gyroscopic couple, $C = I \omega \omega_p$

$= 13.125 \times 994.84 \times 0.1204$

$= 1572.096$ Nm.

Gyroscopic effect:

S.No: 8.

Stability of a four wheel moving in a curved path:

Let,

$m \rightarrow$ Mass of the vehicle in kg.

$W \rightarrow W = mg \rightarrow$ Weight of the vehicle in N

r_w $\rightarrow$ Radius of wheels in 'm'.

$R \rightarrow$ Radius of curvature $(R > r_w)$ in 'm'.

$x \rightarrow$ Width of track in 'm'.

$h \rightarrow$ Distance of center of gravity above the road surface in 'm'.

$I_W \rightarrow$ Mass moment of inertia of each wheels in kgm^2.

$\omega_W \rightarrow$ Angular velocity of wheels (or) velocity of spin in rad/sec.

$I_E \rightarrow$ Mass moment of inertia of the rotating parts of the engine in kgm^2.

$\omega_E \rightarrow$ Angular velocity of rotating parts of the engine rad/sec.

$G \rightarrow$ Gear ratio $= \frac{\omega_E}{\omega_W}$.

$v \rightarrow$ Linear velocity of the vehicle in m/s

$= r_w$

Inorder to determine the reaction of the ground on the vehicle, when the vehicle taking a turn, the following three loads acting on the vehicle should be considered.

1. Weight of the vehicle (W)
2. Reactive gyroscopic couple (c)
3. Centrifugal force (F_c).

1. Reaction due to weight of the vehicle:

Weight on each wheel $= \frac{W}{4} = \frac{mg}{4}$ (downwar

Reaction of ground on each wheel $= \frac{W}{4} = \frac{mg}{4}$ (upwards)

2. Reaction due to gyroscopic couple:

Gyroscopic couple due to the rotating parts of the engine is given by,

$$C_E = \cancel{4}\left[I_E \cdot \omega_E \cdot \omega_p\right].$$

Magnitude of reaction of gyroscopic couple due to rotating parts of the engine is given by,

$$\frac{F}{2} = \frac{C_E}{2b}.$$

Reactive gyroscopic couple due to four wheels is given by, $C_W = 4\left[I_W \cdot \omega_W \cdot \omega_p\right]$.

Magnitude of vertical reaction of reactive gyroscopic couple of ground on each outer wheel

$$\frac{P}{2} = \frac{C_W}{2x} \text{ (upwards)}$$

On each inner wheels, $\frac{P}{2} = \frac{C_W}{2x}$ (downwards)

3) Reaction due to centrifugal effect:

The overturning couple is given by,

$$C_o = F_c \times h = \frac{mv^2}{R} \times h$$

The magnitude of vertical reaction of the overturning couple of ground on each outer wheel,

$$\frac{Q}{2} = \frac{C_o}{2x} = \frac{mv^2}{2Rx} \text{ (upwards)}.$$

on each inner wheel, $\frac{Q}{2} = \frac{C_o}{2x} = \frac{mv^2 h}{2Rx}$ (downwards)

4. Total Reaction:

Let, $R_o \rightarrow$ Vertical reaction on each outer wheel.

$R_i \rightarrow$ Vertical reaction on each inner wheel.

Reaction on the front wheel 1 $= \frac{W}{4} - \frac{P}{2} - \frac{F}{2} - \frac{Q}{2}$

Reaction on front wheel 2 $= \frac{W}{4} + \frac{P}{2} + \frac{Q}{2} - \frac{F}{2}$.

Reaction on rear wheel 3 $= W/4 - P/2 - \frac{Q}{2} + F/2$.

Reaction on rear wheel 4 $= W/4 + P/2 + Q/2 + F/2$.

$$\therefore \text{Total reaction} = \frac{W}{4} - \cancel{\frac{P}{2}} - \cancel{\frac{F}{2}} - \cancel{\frac{Q}{2}} + \frac{W}{4} + \cancel{\frac{P}{2}} + \cancel{\frac{Q}{2}} - \cancel{\frac{F}{2}} + \frac{W}{4} - \cancel{\frac{P}{2}} - \cancel{\frac{Q}{2}} + \cancel{\frac{F}{2}} + \frac{W}{4} + \cancel{\frac{P}{2}} + \cancel{\frac{Q}{2}} + \cancel{\frac{F}{2}}.$$

$$= W.$$

1) A racing car weighs 20 kN. It has a wheel base of 2m, track width 1m and height of CG 300 mm above the ground level and lies midway between the front and rear axle. The engine flywheel rotates at 3000 rpm clockwise when viewed from the front. The moment of inertia of the flywheel is 4 kgm² and moment of inertia of each wheel is 3 kgm². Find the reactions between the wheels and the ground when the car takes a curve of 15 m radius towards right at 30 km/hr, taking into consideration the gyroscopic and centrifug effects. Each wheel radius is 400 mm

Gn:

$W = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

$b = 2 \text{ m}$

$x = 1 \text{ m}$

$h = 300 \text{ mm} = 0.3 \text{ m}.$

$N = 3000$ rpm.

$I_E = 4\ kgm^2$.

$R = 15\ m$.

$I_W = 3\ kgm^2$.

$v = 30\ km/hr$.

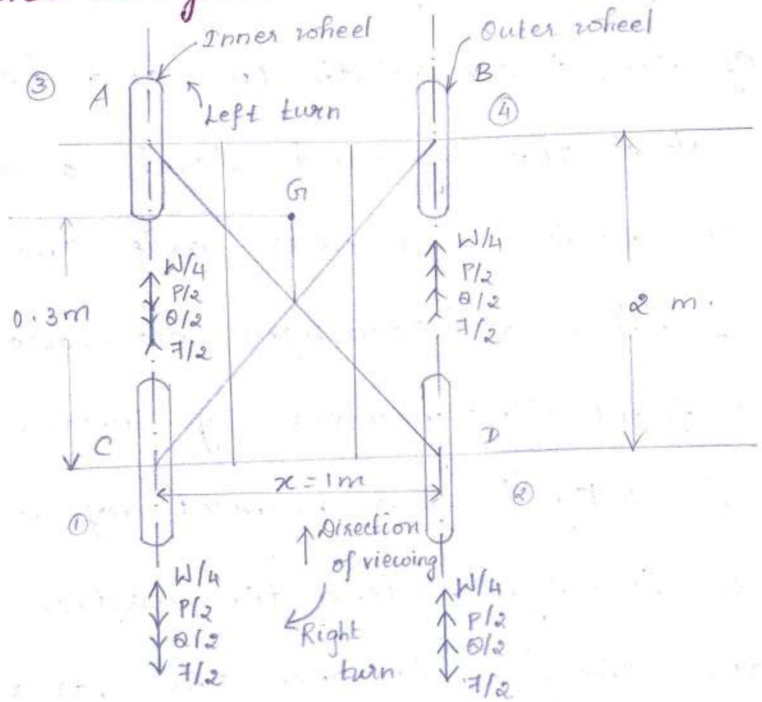
$= \frac{30 \times 10^3}{3600}\ m/s$.

$V = 8.33\ m/s$.

$r_W = 400\ mm = 0.4\ m$.

Soln:

Four wheeler Diagram:



(i) Reaction due to weight of the vehicle:

The weight of the vehicle 'W' will be equally distributed over the four wheels which will act downwards. The reaction between the wheel ... surface for the same magnitude will act upwards.

Load reaction over each wheel $= \frac{W}{4} = \frac{20 \times 10^3}{4}$

$= 5000\ N$ (upwards)

(ii) Reaction due to gyroscopic couple:

W.K.T, angular velocity of the engine flywheel, $\omega_E = \frac{2\pi N}{60}$

$= \frac{2\pi(3000)}{60} = 314.16\ rad/s$

Angular velocity of the wheel, $\omega_W = \frac{V}{r_W}$

$= \frac{8.333}{0.4}$

$\omega_W = 20.83\ rad/s$.

Angular velocity of precession, $\omega_p = \frac{V}{R}$

$= \frac{8.333}{15}$

$\omega_p = 0.555\ rad/s$.

(a) Reaction of gyroscopic couple due to four wheel:

Gyroscopic couple due to four wheel is given by,

$C_W = 4 I_W \omega_W \omega_p$.

$= 4 \times 3 \times 20.83 \times 0.555$.

$C_W = 138.727\ Nm$.

Let us say $\frac{P}{2}$ be the magnitude of gyroscopic couple reaction at each of the inner or outer wheels

$$P/2 = \frac{C_W}{2x}.$$

$$= \frac{138.727}{2 \times 1}$$

$$P/2 = 69.364 \text{ N}$$

Vertical reaction of ground on each outer wheel $= \frac{P}{2} = 69.364$ N (upwards)

Vertical reaction of ground on each inner wheel $= \frac{P}{2} = 69.364$ N (downwards)

(b) Reaction of gyroscopic couple due to rotating parts of engine:

Gyroscopic couple due to rotating parts of the engine is given by,

$$C_E = I_E \omega_E \omega_P ;$$

$$= 4 \times 314.16 \times 0.555$$

$$C_E = 697.44 \text{ Nm}.$$

If $F/2$ is the magnitude of reaction of gyroscopic couple due to rotating parts of engine, which will be vertically downwards on the front (1 & 2) wheels and vertically upwards on the rear wheels (3 & 4)

$$\frac{F}{2} = \frac{C_E}{2b}.$$

$$= \frac{697.44}{2 \times 2}$$

$$\frac{F}{2} = 174.36 \text{ N}$$

Vertical reaction of ground on each front wheel $= \frac{F}{2} = 174.36$ N (upwards)

Vertical reaction of ground on rear wheel $= \frac{F}{2}$
$= 174.36$ (downwards)

(iii) Reaction due to centrifugal couple:

The over turning couple for the car is given by,

$$C_o = F_c \times h.$$

$$= \frac{mv^2}{R} h$$

$$m = \frac{W}{g} = \frac{20 \times 10^3}{9.81} = 2038.736 \text{ N}$$

$$\therefore C_o = \frac{(2038.736)(8.33)^2}{15} \times 0.3$$

$$C_o = 2829.313 \text{ Nm}.$$

Let us say $Q/2$ be the magnitude of reaction due to centrifugal couple on each wheel,

$$\frac{Q}{2} = \frac{C_\theta}{2x}$$

$$= \frac{2829.313}{2 \times 1}$$

$$\frac{Q}{2} = 1415 \text{ N}$$

Vertical reaction of ground on each of the wheel $= \frac{Q}{2} = 1415$ (upwards)

Vertical reaction of ground on each inner wheel $= \frac{Q}{2} = 1415$ (downwards)

iv) Total reactions:

From fig.

Reaction on front wheel ① $= \frac{W}{4} - \frac{P}{2} - \frac{F}{2} - \frac{Q}{2}$.

$= 5000 - 69.364 - 174.36 - 1415$.

$= 3341.276$ N.

Reaction on front wheel ② $= \frac{W}{4} + \frac{P}{2} - \frac{F}{2} + \frac{Q}{2}$

$= 5000 + 69.364 - 174.36 + 1415$

Reaction on rear wheel ③ $= \frac{W}{4} - \frac{P}{2} + \frac{F}{2} - \frac{Q}{2}$

$= 5000 - 69.364 + 174.36 - 1415$.

$= 3689.99$ N.

Reaction on rear wheel ④ $= \frac{W}{4} + \frac{P}{2} + \frac{F}{2} + \frac{Q}{2}$.

$= 5000 + 69.364 + 174.36 + 1415$

$= 6658.724$ N.

Stability of a two wheel vehicle moving in a curved path:

Like a four wheel vehicle, two wheel vehicle will also be subjected to gyroscopic and centrifugal couple when it takes a turn on a curved path.

Let us consider a two wheel vehicle taking a right turn.

Let,

$m \rightarrow$ mass of the vehicle in Kg.

$W = mg \rightarrow$ Weight of the vehicle in N

$r_W \rightarrow$ Radius of the wheels in 'm'.

$R \rightarrow$ Radius of track or curvature in 'm'.

$h \rightarrow$ Height of the center of gravity of the vehicle in 'm'.

$I_W \rightarrow$ Mass moment of inertia of each wheel in $Kg\,m^2$.

$I_E \rightarrow$ Mass moment of inertia of the

$\omega_W \rightarrow$ Angular velocity of wheel in rad/sec $= \frac{v}{r_W}$

$\omega_E \rightarrow$ Angular velocity of the engine in rad/sec.

$\omega_P \rightarrow$ Angular velocity of precession in rad/sec $= \frac{v}{R}$.

$G \rightarrow$ Gear ratio $= \frac{\omega_E}{\omega_W}$

$v \rightarrow$ Linear velocity of the vehicle in m/sec

$\theta \rightarrow$ Angle of heel in degrees.

2m:

Angle of heel :

The angle of inclination of the vehicle to the vertical is known as the angle of heel.

Pbm:

1) Each road wheel of a motor cycle has a mass moment of inertia of 1.5 kgm². The rotating parts of the engine of the motor cycle have a mass moment of inertia of 0.25 kgm². The speed of the engine is 5 times the speed of wheels and it is in the same sense. The mass of the motor cycle with it's rider is 250kg and it's center of gravity is 0.6 m above the ground level. Find the angle of heel if the ... is travelling at 50 kmph and is taking a

Gn:

$I_W = 1.5 \text{ kgm}^2$

$I_E = 0.25 \text{ kgm}^2$

$\frac{\omega_E}{\omega_W} = G = 5$

$m = 250$ kg

$h = 0.6$ m

$v = 50 \text{ km/hr} = \frac{50 \times 10^3}{3600} \text{ m/s} = 13.89 \text{ m/s}.$

$R = 30$ m

$d_W = 0.6$ m

(or)

$r_W = \frac{0.6}{2} = 0.3$ m.

Soln:

W.K.T, Gyroscopic couple due to two wheels and due to rotating parts of the engine

$$C_1 = \frac{v^2}{r_W R}(2I_W + GI_E)\cos\theta.$$

$$= \frac{(13.89)^2}{(0.3)(30)}\left[2(1.5) + 5(0.25)\right]\cos\theta.$$

$$C_1 = 91.09 \cos\theta \text{ Nm}$$

The couple due to centrifugal force acting on the cycle is given by, $C_2 = \left(\frac{mv^2}{R}\right)h\cos\theta$

$$= \frac{250\,(13.89)^2}{30}(0.6)\cos\theta$$

$$C_2 = 964.66\cos\theta \text{ Nm}$$

Total overturning couple, $C_\theta = C_1 + C_2$.

$$C_\theta = (91.09 + 964.66)\cos\theta$$

$$C_\theta = 1055.75 \cos\theta \text{ Nm.}$$

W.K.T, Balancing couple $= mgh \sin\theta$.

$$= 250 \times 9.81 \times 0.6 \sin\theta$$

$$= 1471.5 \sin\theta \text{ Nm}.$$

For stability,

Overturning couple = Balancing couple

$$1055.75 \cos\theta = 1471.5 \sin\theta.$$

$$\frac{\sin\theta}{\cos\theta} = 0.717$$

$$\Rightarrow \tan\theta = 0.717$$

$$\therefore \theta = 35.65^\circ.$$

$\therefore$ Angle of heel, $\theta = 35.65^\circ$